
Research article

On The Algebraic Structure of neutrosophic Ideal ν -Open Sets in Topological Spaces

Zaman T. Samer^{1,*}

¹ Salahaddin Education Directorate, 34001 Salahaddin, Iraq.

* **Correspondence:** zaman.t.sameer@st.tu.edu.iq

ARTICLE INFO

Keywords:

Neutrosophic
 ν -Open Sets
topological structure

Mathematics Subject Classification:

03C05, 08A05

Important Dates:

Received: 12 December 2025

Revised: 31 December 2025

Accepted: 1 January 2026

Online: 2 January 2026



Copyright © 2025 by the authors. Published
under Creative Commons Attribution (CC
BY) license.

ABSTRACT

This work describes a unified study of multi-topological topological structures and neutrosophic (neutr.) algebraic structures in accordance with the concept of the ideal to provide a new form of mathematics to accommodate uncertainty and multi-composition at once. The paper starts by generalizing open sets via the theory of ν -open sets and then integrates this concept with neutr. logic, that is to say, the interplay between truth, indeterminacy and falsity being its independent constituents. The integration is not a mere descriptive thing, but being explicitly formulated by expressing neutr. open sets and directly linking to a neutr. ideal in the neutr. loop, for purposes of algebraic control over the topological structure. The study offers clear, simple definitions at the fundamental level, then introduces internal concepts such as neutr. closure, inside, outside, limit, neighbor and accumulation points to illustrate the important properties in new perspective of the new framework. Most importantly, you observe the retention of the fundamental topological axioms in ideal neutr. spaces and how these are radically different from classical topology, specifically relating that to the effect of the indeterminacy complex and the function of the algebraic ideal in change of the local behavior of groups.

1. Introduction

After the presentation of open sets in topological spaces (T.P) , each studies were presented for new types of open sets, some of which are based on each basic topological spaces, to mention but not limited to: in 2018 and showed them usefully and their properties, in addition to open sets that were used to build graph

models, and clarify the topological structure of the distribution of these graphs into subsets by [7], in 2012 by [8], and the upper semi-continuous open topologies presented by [6], open sets $(\wedge, b)$ presented by [16] in 2022, and $\ast$ -open sets, and at the same time $(\delta$ -open) were presented by [17]. These open sets helped each researchers to understand and present new topological spaces and explain their properties. Sharma et al. first introduced the quadrupole topology and developed it as in [12], where a new open set termed q-open was constructed and relied on. Many studies on these topologies have been presented, including fuzzy q-open sets and fuzzy q-b-open sets, as well as q-cont and q-b-cont functions. Beyond introducing a new function for the quadrupole topology space in [9], they designed completely new and different kinds of sets [15]. Moreover, Pang et al. proposed a topological framework of a quadrupole transmitter constituted by a first topology and a second topology, the first topology comprising a first bridge, a second phase shift element, and a third phase shift element [10]. These studies collectively offer insights into various topologies and their properties.

The interplay between algebraic structures and topological spaces has been a fertile ground for mathematical research, yielding profound insights and applications across various domains. Classical topology, initiated by the fundamental concept of open sets, has evolved through numerous generalizations including semi-open sets, pre-open sets, α -open sets, and b-open sets, among others. More recently, the notion of ν -open sets has emerged as a significant generalization [11], defined in the context of multiple topologies on a common universe. These sets—characterized by the existence of a non-empty set common to all topologies—provide a robust framework for studying spaces with multiple compatible topological structures. Concurrently, neutr. logic, pioneered by Smarandache, has revolutionized uncertainty modeling by extending fuzzy and intuitionistic fuzzy logics through three independent components: truth (T), indeterminacy (I), and falsity (F) [13]. This tripartite structure has found rich applications in algebraic contexts through neutr. groups, rings, semigroups, and ideals. Neutr. algebraic structures incorporate the indeterminate element I, allowing for the mathematical representation of ambiguous, incomplete, or contradictory information.

Despite significant advancements in both domains, a crucial gap remains: the integration of neutr. algebraic ideals with generalized topological structures. While neutr. topological spaces have been explored, and various generalizations of open sets have been studied independently, the specific combination of neutr. ideals with ν -open set theory represents unexplored mathematical territory. This integration is particularly compelling because Algebraic-topological synthesis: Combining the algebraic precision of ideals with the spatial intuition of topology creates a powerful framework for structured uncertainty. with multi-topological uncertainty: Real-world systems often operate under multiple topological perspectives simultaneously, each with inherent indeterminacy and Ideal-based perturbations: The concept of ideals naturally models "small" or "negligible" elements, allowing for controlled modifications of topological structures.

2. Fundamental Concepts

Definition 2.1. [11] Let $\{\tau_k\}_{k \in J}$, $k \geq 2$ be a collection of a topological space on X , and $\kappa \subseteq X$. Then κ is termed ν -open set (ν_O -set) in X if any $T \in \bigcap_{k \in J} \tau_k$ under which $\emptyset \neq T \subseteq \kappa$ where $\kappa \neq \emptyset$ and $\emptyset = T$ where $\kappa = \emptyset$.

Definition 2.2. [11] The set of all ν_O -set in X (ν_{oX}) and define as $\nu_{oX} = \{\kappa : \kappa \text{ is } \nu\text{-open in } X\}$. If $A \subseteq X$ a point $a \in A$ is termed interior A in X if and only if there exist $U_x \in \nu_{oX}$ under which $a \in U_x \subseteq A$ then $\text{int}_\nu = \bigcup_{i \in \mathbb{Z}} \{x_i : x_i \text{ is } \nu\text{-open}, x_i \in A \forall i \in I\}$. The complement of a ν -open is termed ν -closed set (ν_C -set) and

the set of all v_C -set is denoted by CS_v . An order pair (X, v_{oX}) is termed v -space where X is a universal set and v_{oX} is set of all v_O -set in X .

Definition 2.3. [11] Let (X, v_{oX}) be a v -space. A v -closure of $\kappa \subseteq X$ ($cl_v(\kappa)$) its intersection of all v_C -set that include κ .

Theorem 2.4. [11] Let (X, v_{oX}) be a v -space and $\kappa \subseteq X$, then κ is a v_C -set that include κ .

Corollary 2.5. [11] Assume (X, v_{oX}) is a v -space and $\kappa \subseteq X$, then κ is a v_C -set if and only if

$$\kappa = cl_v(\kappa).$$

Remark 2.6. [11] Let $\{\tau_k\}_{2 \leq k \in J}$ be a family of topological space on X and τ be a proper non-empty set of X under which $\tau \in \tau_k$, for some $k \in J$. However, this is not required $\tau \in v_{oX}$.

Remark 2.7. [11] Let $\{\tau_k\}_{k \in J}$ be a family of topological space on X and $\kappa \in v_{oX}$. However, this is not required $\kappa \in \tau_k$ for all $k \in J$.

Proposition 2.8. [11] Let $\{\tau_k\}_{k \in J}$ be a family of topological space on X . If $\kappa \in \tau_k$ for all $k \in J$, then $\kappa \in v_{oX}$. In general, the converse is not true.

Definition 2.9. Neutr. logic generalizes fuzzy and intuitionistic fuzzy logics by evaluating properties P via three independent components:

$$P = \langle T, I, F \rangle$$

where

- T as the membership of truth (certainty),
- I as the membership of indeterminacy (ambiguity),
- F as the membership of falsity (negativity).

Definition 2.10. [13] A neutr. number is an element in a single-valued neutr. set, represent as $\langle T, I, F \rangle$.

Example 2.11. $x(I) = 5 + \sqrt{2}I$ has a determinate part 5 and in determinate part $\sqrt{2}I$.

Definition 2.12. Let S be a universal set. A neutr. set K is defined as form:

$$K = \{(\lambda, T_K(\lambda), I_K(\lambda), F_K(\lambda)) \mid \lambda \in S\}$$

where $T_K, I_K, F_K : S \rightarrow [0, 1]$, represent truth, indeterminacy, and falsity membership functions, respectively.

Definition 2.13. [18] Let T, I, F be standard or non-standard of $[0, 1]$:

$$\begin{aligned} \sup T &= t_{\sup}, \inf T &= t_{\inf} \\ \sup I &= i_{\sup}, \inf I &= i_{\inf} \\ \sup F &= f_{\sup}, \inf F &= f_{\inf} \\ n_{\sup} &= t_{\sup} + i_{\sup} + f_{\sup} \\ n_{\inf} &= t_{\inf} + i_{\inf} + f_{\inf} \end{aligned}$$

where T, I, F are termed neutr. components.

Definition 2.14. [2] If $(H, *)$ be a group and let $(H \cup I) = \{n + mI : n, m \in H\}$. $N(H) = ((G \cup I), *)$ is neutr. group by H and I with the binary operation $*$. I is termed the neutr. element, $I^2 = I$.

For example: $(N(\mathbb{C}), +)$, $(N(\mathbb{Q}), +)$, and $(N(\mathbb{Z}), +)$ are neutr. groups.

Definition 2.15. [14] Let $(S, .)$ be a semigroup. The neutr. semigroup $S(I) = \langle S \cup I \rangle$ is an algebraic structure generated by S and the indeterminate I . A neutr. subsemigroup $T(I)$ of $S(I)$ is a subset closed under the semigroup operation and containing I .

Definition 2.16. [3] Let $(G, *)$ be a group, and let $G(I) = \langle G \cup I \rangle$ be a neutr. group generated by G and an indeterminate element I . A subset $H(I)$ of $G(I)$ is termed neutr. subgroup if $H(I)$ is neutr. group under operation $*$, the restriction of $H(I)$ to G is a classical subgroup of G .

Definition 2.17. [4] If $(R, +, .)$ be a ring. Then the set $(R \cup I) = \{c + dI : c, d \in R\}$ is termed a neutr. ring generated by R and I under the operations of R . For example: $\mathbb{Z}(I) = \{a + bI \mid a, b \in \mathbb{Z}\}$.

Definition 2.18. [5] Let $R(I)$ be a neutr. ring, where R is a classical ring and I is an indeterminate. A non-empty subset $S(I)$ of $R(I)$ is termed a neutr. subring if $S(I)$ is a neutr. ring under the same operations, and $\cap S = S(I) \cap R$ is a classical subring of R .

Definition 2.19. [1] Let $R(I)$ be a neutr. ring, where R is a classical ring and I is an indeterminate. A non-empty subring $J(I)$ of $R(I)$ is termed a neutr. ideal if for every $r(I) \in R(I)$ and $j(I) \in J(I)$ both $r(I) \cdot j(I)$ and $j(I) \cdot r(I)$ belong to $J(I)$.

3. Neutr. v -Open Set and Neutr. v -Ideal

This section introduces and develops the theory of neutr. v -open sets and their enhancement through algebraic ideals to form neutr. ideal v_O -sets. We establish:

Definition 3.1. Let $\{\tau_k\}_{2 \leq k \in J}$ be a collection of two or more topological spaces on the same universe X . Let $NS(X)$ be the family of all neutr. sets over X , where a neutr. set A is of the form:

$$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X$$

with $T_A, I_A, F_A : X \rightarrow [0, 1]$.

A neutr. set $\eta \in NS(X)$ is termed a neutr. v -open set (Nv_O -set) if there exists a non-empty crisp v_O -set $U \in v_{OX}$, i.e., $U \in \bigcap_{k \in J} \tau_k$, $U \neq \emptyset$ under which: $U \subseteq Supp_T(\eta)$, and we define the neutr. membership for η based on U in the following: way:

$$\begin{aligned} \bullet \text{ For } x \in U &\implies \begin{cases} T_\eta(x) = 1 \\ I_\eta(x) = 0 \\ F_\eta(x) = 0 \end{cases} \\ \bullet \text{ For } x \notin U &\implies \begin{cases} T_\eta(x) = 0 \\ I_\eta(x) = 1 \\ F_\eta(x) = 1 \end{cases} \end{aligned}$$

Alternatively, more generally, we can say: η is neutr. v_O -set if any a crisp v_O -set U under which η coincides with the neutr. characteristic function of U , i.e.: $\eta = \langle \chi_U, 0, 1 - \chi_U \rangle$, where χ_U is the crisp characteristic function of U (1 if $x \in U$, 0 otherwise), so: $T_\eta = \chi_U$, $I_\eta = 0$, $F_\eta = 1 - \chi_U$.

Example 3.2. Let $X = \{a, b, c\}$. Consider two topologies on X : $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then: $\bigcap_{k=1,2} \tau_k = \{\emptyset, \{a\}, X\}$. So the crisp v_O -sets are: $\emptyset, \{a\}, X$. Take $U = \{a\}$ (crisp v_O -set).

Define the neutr. set τ corresponding to U :

- For $x = a$: $T = 1, I = 0, F = 0$
- For $x = b$: $T = 0, I = 1, F = 1$
- For $x = c$: $T = 0, I = 1, F = 1$

So: $\eta = \{(a, 1, 0, 0), (b, 0, 1, 1), (c, 0, 1, 1)\}$. This η is a Nv_O -set.

Definition 3.3. Let $R(I)$ be a neutr. ring. A non-empty subset $J(I) \subseteq R(I)$ is a neutr. ideal of $R(I)$ (Nv_I) if: $J(I)$ is a neutr. subring (closed under subtraction and contains I), and For every $r(I) \in R(I)$ and $j(I) \in J(I)$, both $r(I) \cdot j(I) \in J(I)$ and $j(I) \cdot r(I) \in J(I)$.

Example 3.4. Let $R = Z_2 = \{0, 1\}$ (ring mod 2). The neutr. ring: $R(I) = \{0, 1, I, 1 + I\}$, $I^2 = I$. Take $X = \{1, 2, 3\}$, with $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then: $\bigcap_{k=1,2} \tau_k = \{\emptyset, \{a\}, X\}$, hence its v_O -sets.

Take $U = \{a\}$. Now we know the neutr. group η corresponding to U as $\eta = \{(a, 1, 0, 0), (b, 0, 1, 1), (c, 0, 1, 1)\}$, where $\eta(a) = (T = 1, I = 0, F = 0)$, $\eta(b) = (T = 0, I = 1, F = 1)$, and $\eta(c) = (T = 0, I = 1, F = 1)$, take $J(I) = \{0, I\}$, then we get: $I \in J(I)$, $0 + I = I \in J(I)$, $I + I = 0 \in J(I)$, $I \cdot I = I \in J(I)$, $0 \cdot I = 0 \in J(I)$.

To define Nv_I take $\xi(I)$ is Nv_I on $J(I)$ define by form:

$$\xi(a) = (0, I, 0)$$

$$\xi(b) = (I, 0, I)$$

$$\xi(c) = (0, I, I)$$

Now we know $v = \eta + \xi$ (where the sum is the plural of neutrosophy):

$$v(a) = (1 + 0, 0 + I, 0 + 0) = (1, I, 0)$$

$$v(b) = (0 + I, 1 + 0, 1 + I) = (I, 1, 1 + I)$$

$$v(c) = (0 + 0, 0 + 1, 1 + I) = (0, 1, 1 + I)$$

Then $v = \{(a, 1, I, 0), (b, I, 1, 1 + I), (c, 0, 1, 1 + I)\}$ is Nv_I since η is a Nv_O -set, ξ takes its values from the neutr. example $J(I)$, and $v = \eta + \xi$.

Definition 3.5. Let X be a set with two or more topologies $\tau_1, \tau_2, \dots, \tau_n$, giving $(X, \tau_1, \tau_2, \dots, \tau_n)$ -set structure.

Let $N(X)$ be the collection of neutr. X -sets. Let I be a neutr. ideal in some neutr. ring R . We say a neutr. set $A \in N(X)$ is a neutr. ideal I -open set (I -set) with respect to R if for any $U \in \tau_i$, and B is a neutr. set whose membership values $\mu_B(x), \sigma_B(x), \nu_B(x)$ are elements of the neutr. ideal I for each $x \in X$,

i.e., $A = U \oplus B$ with $B \in \mathcal{I}$, etc., but in practice for single-valued neutr. sets we take $\mu_B, \sigma_B, \nu_B \in \mathcal{I}$, so we interpret: the triple (μ_A, σ_A, ν_A) belongs to some subset of $\mathcal{N}(X)$ that is isomorphic to a neutr. ideal in a formal way, then $A \in \mathcal{I}$. More concretely, we can define: Let I be an ordinary ideal in a ring R . Define a neutr. ideal $\mathcal{I} = \{(n_1, n_2, n_3) : n_1, n_2, n_3 \in I\}$. A neutr. set A is termed $\mathcal{I}$ -set if for each $x \in X$, $\mu_A(x), \sigma_A(x), \nu_A(x) \in I$, and $A = U \cup B$ where U is τ_i -set, and $B \in \mathcal{I}$ (neutr. numbers from the ideal).

Example 3.6. Take $X = \{a, b, c\}$, with $\tau_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{b\}, \{a, b\}, X\}$. Then: $\bigcap_{k=1,2} \tau_k = \{\emptyset, \{a, b\}, X\}$, hence its ν_O -sets. Take $U = \{a, b\}$. Now we know the neutr. group η corresponding to U as $\eta = \{(a, 1, 0, 0), (b, 1, 0, 0), (c, 0, 1, 1)\}$, where $\eta(a) = (T = 1, I = 0, F = 0)$, $\eta(b) = (T = 1, I = 0, F = 0)$, and $\eta(c) = (T = 0, I = 1, F = 1)$.

Let $R = \mathbb{Z}_2 = \{0, 1\}$ (ring mod 2). The neutr. ring: $R(I) = \{0, 1, I, 1 + I\}$, $I^2 = I$, take $J(I) = \{0, I\}$.

To define Nv_I take $\xi(I)$ is Nv_I on $J(I)$ define by form:

$$\xi(a) = (0, I, 0)$$

$$\xi(b) = (I, 0, 0)$$

$$\xi(c) = (0, 0, I)$$

Now we know $v = \eta + \xi$ (where the sum is the plural of neutr.):

$$v(a) = (1 + 0, 0 + I, 0 + 0) = (1, I, 0)$$

$$v(b) = (1 + I, 0 + 0, 0 + 0) = (1 + I, 0, 0)$$

$$v(c) = (0 + 0, 0 + 1, 1 + I) = (0, 1, 1 + I)$$

Then $v = \{(a, 1, I, 0), (b, 1 + I, 0, 0), (c, 0, 1, 1 + I)\}$ is $N_I\nu_O$ -set.

Remark 3.7. Define the family of Nv_O -set, s.t. $Nv_O(X) \subseteq NS(X)$ as follows: for each crisp ν_O -set $U \in \tau_\nu$, its corresponding neutr. ν_O -set $\eta_U \in Nv_O(X)$ is defined by:

$$\eta_U(x) = \begin{cases} (1, 0, 0) & \text{if } x \in U \\ (0, 1, 1) & \text{if } x \notin U \end{cases}$$

Definition 3.8. define the collection

$$\tau_{Nv_I} = \{v \in NS(X) \mid v \text{ is } N_I\nu_O\text{-set}\}$$

The pair $(X, \mathcal{I})$ is termed a neutr. ideal $\mathcal{I}$ -open topological space ($\mathcal{I}$ -TS), if it satisfies the topological axioms:

1) $\emptyset_I, X_I \in \tau_I$, where $\emptyset_I$ is the empty neutr. set corresponding to the crisp $\emptyset$, with $\mu = 0, \sigma = 0, \nu = 1$. While X_I is the space neutr. set corresponding to the crisp X , with $\mu = 1, \sigma = 1, \nu = 0$.

2) Let $\{A_j\}_{j=1}^n \subseteq \tau_I$ with $A_j = U_j \oplus B_j$, $U_j \in \tau_i, B_j \in \mathcal{I}$. Then:

$$\bigcap_{j=1}^n A_j = \bigcap_{j=1}^n (U_j \oplus B_j) = \left(\bigcap_{j=1}^n U_j \right) \oplus \left(\bigcup_{j=1}^n B_j \right)$$

Since $\bigcap U_j \in \tau_i$ (as crisp τ_i -sets are closed under finite intersections) and $\bigcup B_j \in \mathcal{I}$ (as $\mathcal{I}$ is closed under addition), we have $\bigcap A_j \in \tau_I$.

3) Let $\{A_j\}_{j \in J} \subseteq \tau_I$ with $A_j = U_j \oplus B_j$. Then:

$$\bigcup_{j \in J} A_j = \bigcup_{j \in J} (U_j \oplus B_j) = \left(\bigcup_{j \in J} U_j \right) \oplus \left(\bigcup_{j \in J} B_j \right)$$

Since $\bigcup U_j \in \tau_i$ and $\bigcup B_j \in \mathcal{I}$ (as $\mathcal{I}$ is closed under arbitrary sums), we have $\bigcup A_j \in \tau_I$.

4. Some Properties for Neutr. v -Open Set

Definition 4.1. The neutr. closure of a neutr. set v in N_{IV_O} -space is the smallest of N_{IV_C} -set containing v . Formally:

$$Ncl_{(N_{IV})}(v) = \bigcap \{ \omega \in NS(X) \mid \omega^c \in \tau_{(N_{IV})}, v \subseteq \omega \}$$

Example 4.2. Let $X = \{a, b, c\}$, with two topologies: $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then $\tau_v = \{\emptyset, \{a\}, X\}$. Let $R = \mathbb{Z}_2$, $J = \{0\} \subseteq R$, so $J(I) = \{0, I\}$. Take $v = \{(a, 1, I, 0), (b, 0, 1, 1), (c, 0, 1, 1)\}$ (corresponds to crisp $\{a\}$). Then

$$Ncl_{(N_{IV})}(v) = v \cup \{(b, I, 0, 0), (c, 0, 0, 1)\} = \{(a, 1, I, 0), (b, I, 1, 1), (c, 0, 1, 1 + I)\}$$

(smallest N_{IV_C} -set containing v).

Definition 4.3. The neutr. interior of a neutr. set v in N_{IV_O} -space is the union of N_{IV_C} -set containing v . Formally:

$$Nint_{(N_{IV})}(v) = \bigcup \{ \omega \in \tau_{(N_{IV})} \mid \omega \subseteq v \}$$

Example 4.4. Let $X = \{a, b, c\}$, with two topologies: $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then $\tau_v = \{\emptyset, \{a\}, X\}$. Let $R = \mathbb{Z}_2$, $J = \{0\} \subseteq R$, so $J(I) = \{0, I\}$. Take $v = \{(a, 1, I, 0), (b, 1 + I, 1, 1), (c, 0, 1, 1 + I)\}$. Then

$$Nint_{(N_{IV})}(v) = \{(a, 1, 0, 0), (b, 1, 0, 0)\}, \text{ (corresponds to crisp } \{a, b\} \text{ which is } v_O\text{-set).}$$

Definition 4.5. The neutr. exterior of a neutr. set v is the interior of its complement Formally:

$$Next_{(N_{IV})}(v) = Nint_{(N_{IV})}(v^c)$$

where $v^c(x) = (F_v(x), I_v(x), T_v(x))$ for each $x \in X$.

Example 4.6. Let $X = \{a, b, c\}$, with two topologies: $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then $\tau_v = \{\emptyset, \{a\}, X\}$. Let $R = \mathbb{Z}_2$, $J = \{0\} \subseteq R$, so $J(I) = \{0, I\}$. Take $v = \{(a, 1, I, 0), (b, 0, 1, 1), (c, 0, 1, 1)\}$. Then $v^c = \{(a, 0, I, 1), (b, 1, 1, 0), (c, 1, 1, 0)\}$

$$Next_{(N_{IV})}(v) = \{(b, 1, 0, 0), (c, 1, 0, 0)\}, \text{ (corresponds to crisp } \{b, c\} \text{ which is } v_O\text{-set).}$$

Definition 4.7. The neutr. boundary of a neutr. set v is

$$Nbd_{(N_{IV})}(v) = \frac{Ncl_{(N_{IV})}(v)}{Nint_{(N_{IV})}(v)}$$

Example 4.8. Let $X = \{a, b, c\}$, with two topologies: $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then $\tau_v = \{\emptyset, \{a\}, X\}$. Let $R = \mathbb{Z}_2$, $J = \{0\} \subseteq R$, so $J(I) = \{0, I\}$. Take $v = \{(a, 1, I, 0), (b, 1 + I, 0, 0), (c, 0, 1, 1)\}$. $Ncl_{(N_I v)}(v) = v \cup \{(c, I, 0, 0)\} = \{(a, 1, I, 0), (b, 1 + I, 0, 0), (c, I, 1, 1)\}$. $Nint_{(N_I v)}(v) = \{(a, 1, 0, 0), (b, 1, 0, 0)\}$. $Nbd_{(N_I v)}(v) = \{(a, 0, I, 0), (b, I, 0, 0), (c, I, 1, 1)\}$.

Definition 4.9. A neutr. set ω is termed a $N_I v$ -neighborhood ($N_I v$ -neigh) of a point $x \in X$ if there exists $\eta \in \tau_{(N_I v)}$ under which: $\eta(x) = (1, 0, 0)$ and $\eta \subseteq \omega$.

Example 4.10. For $x = a$, the neutr. set $\eta = \{(a, 1, 0, 0), (b, 0, 1, 1), (c, 0, 1, 1)\}$ is a $N_I v_O$ -set. Any ω with $\eta \subseteq \omega$ is a $N_I v$ -neigh of a .

Definition 4.11. A point $x \in X$ is a $N_I v$ -limit point of neutr. set v if for every $N_I v$ -neigh η of x , there exists $y \neq x$ under which: $T_\eta(y) > 0$ and $T_v(y) > 0$. The set of all $N_I v$ -limit points of v is denoted $Nd_{N_I v}(v)$.

Example 4.12. Let $v = \{(a, 0.8, 0.2, 0.1), (b, 0.7, 0.3, 0.2), (c, 0, 1, 1)\}$. For $x = a$, consider $N_I v$ -neigh $\eta = \{(a, 1, 0, 0), (b, 0.5, 0.5, 0), (c, 0, 1, 1)\}$. Since $T_\eta(b) = 0.5 > 0$ and $T_v(b) = 0.7 > 0$, $a \in Nd_{N_I v}(v)$.

Remark 4.13. In $N_I v$ -spaces, limit points depend on both the v -open structure and the neutr. ideal perturbations. Unlike classical topology where only truth membership matters, here indeterminacy and falsity components from $J(I)$ affect neighborhood structures.

Theorem 4.14. Let $(X, \tau_{(N_I v)})$ be a neutr. ideal v -open space. For any $v_1, v_2 \in NS(X)$, then:

- $Nint_{(N_I v)}(\emptyset_N) = \emptyset_N$ and $Nint_{(N_I v)}(X_N) = X_N$
- If $v_1 \subseteq v_2 \implies Nint_{(N_I v)}(v_1) \subseteq Nint_{(N_I v)}(v_2)$.
- $Nint_{(N_I v)}(v_1 \cap v_2) = Nint_{(N_I v)}(v_1) \cap Nint_{(N_I v)}(v_2)$.
- $Nint_{(N_I v)}(v_1 \cup v_2) \supseteq Nint_{(N_I v)}(v_1) \cup Nint_{(N_I v)}(v_2)$
- v is $N_I v_O$ -set $\iff Nint_{(N_I v)}(v) = v$.

Proof. **1. For $Nint_{(N_I v)}(\emptyset_N)$ and $Nint_{(N_I v)}(X_N)$:** By definition of $Nint_{(N_I v)}(\emptyset_N)$, $\emptyset_N$ is trivially a $N_I v_O$ -set. Thus, its $N_I v_O$ -set is itself. For X_N , $Nint_{(N_I v)}(X_N) = X_N$. The idempotence follows because $N_I v_O$ -sets are preserved under interior operations.

Let $\mathcal{F}_1 = \{\eta \in \tau_{(N_I v)} \mid \eta \subseteq v_1\}$ and $\mathcal{F}_2 = \{\eta \in \tau_{(N_I v)} \mid \eta \subseteq v_2\}$. Since $v_1 \subseteq v_2$, any η that satisfies $\eta \subseteq v_1$ automatically satisfies $\eta \subseteq v_2$. Thus, $\mathcal{F}_1 \subseteq \mathcal{F}_2$. Taking unions:

$$Nint_{(N_I v)}(v_1) = \bigcup \mathcal{F}_1 \subseteq \bigcup \mathcal{F}_2 = Nint_{(N_I v)}(v_2)$$

Therefore, $Nint_{(N_I v)}(v_1) \subseteq Nint_{(N_I v)}(v_2)$.

We prove both inclusions for $Nint_{(N_I v)}(v_1 \cap v_2) = Nint_{(N_I v)}(v_1) \cap Nint_{(N_I v)}(v_2)$.

($\subseteq$) Show $Nint_{(N_I v)}(v_1 \cap v_2) \subseteq Nint_{(N_I v)}(v_1) \cap Nint_{(N_I v)}(v_2)$: Since $(v_1 \cap v_2) \subseteq v_1$ and $(v_1 \cap v_2) \subseteq v_2$, by monotonicity (Part 2): $Nint_{(N_I v)}(v_1 \cap v_2) \subseteq Nint_{(N_I v)}(v_1)$ and $Nint_{(N_I v)}(v_1 \cap v_2) \subseteq Nint_{(N_I v)}(v_2)$. Thus, $Nint_{(N_I v)}(v_1 \cap v_2) \subseteq Nint_{(N_I v)}(v_1) \cap Nint_{(N_I v)}(v_2)$.

($\supseteq$) Show $Nint_{(N_I v)}(v_1) \cap Nint_{(N_I v)}(v_2) \subseteq Nint_{(N_I v)}(v_1 \cap v_2)$: Let η be any $N_I v_O$ -set such that $\eta \subseteq Nint_{(N_I v)}(v_1) \cap Nint_{(N_I v)}(v_2)$. Then $\eta \subseteq Nint_{(N_I v)}(v_1)$ and $\eta \subseteq Nint_{(N_I v)}(v_2)$. By definition of interior, since η is a $N_I v_O$ -set and $\eta \subseteq Nint_{(N_I v)}(v_1) \subseteq v_1$, we have $\eta \subseteq v_1$. Similarly, $\eta \subseteq v_2$. Thus, $\eta \subseteq v_1 \cap v_2$. Therefore,

η is a $N_I\nu_O$ -set contained in $\nu_1 \cap \nu_2$, so $\eta \subseteq Nint_{(N_I\nu)}(\nu_1 \cap \nu_2)$. Taking the union over all such η gives: $Nint_{(N_I\nu)}(\nu_1 \cap \nu_2) \supseteq Nint_{(N_I\nu)}(\nu_1) \cap Nint_{(N_I\nu)}(\nu_2)$.

Since $\nu_1 \subseteq \nu_1 \cup \nu_2$ and $\nu_2 \subseteq \nu_1 \cup \nu_2$, by monotonicity (Part 2): $Nint_{(N_I\nu)}(\nu_1) \subseteq Nint_{(N_I\nu)}(\nu_1 \cup \nu_2)$ and $Nint_{(N_I\nu)}(\nu_2) \subseteq Nint_{(N_I\nu)}(\nu_1 \cup \nu_2)$. Thus, $Nint_{(N_I\nu)}(\nu_1) \cup Nint_{(N_I\nu)}(\nu_2) \subseteq Nint_{(N_I\nu)}(\nu_1 \cup \nu_2)$.

($\Rightarrow$) Assume ν is a $N_I\nu_O$ -set. By definition, $\nu \in \tau_{(N_I\nu)}$, so ν is a $N_I\nu_O$ -set contained in itself. Thus, $\nu \in \{\eta \in \tau_{(N_I\nu)} \mid \eta \subseteq \nu\}$. Since $Nint_{(N_I\nu)}(\nu)$ is the union of all such sets, we have $Nint_{(N_I\nu)}(\nu) = \nu$.

($\Leftarrow$) Assume $Nint_{(N_I\nu)}(\nu) = \nu$. By definition of interior: $\nu = \bigcup \{\eta \in \tau_{(N_I\nu)} \mid \eta \subseteq \nu\}$. The union of $N_I\nu_O$ -sets is a $N_I\nu_O$ -set (as $\tau_{(N_I\nu)}$ is closed under arbitrary unions). Therefore, ν is a $N_I\nu_O$ -set. $\square$

Theorem 4.15. *Let $(X, \tau_{(N_I\nu)})$ be a neutr. ideal ν -open space. For any $\nu_1, \nu_2 \in NS(X)$, then:*

- $Ncl_{(N_I\nu)}(\emptyset_N) = \emptyset_N$ and $Ncl_{(N_I\nu)}(X_N) = X_N$, and $Ncl_{(N_I\nu)}(Ncl_{(N_I\nu)}(\nu)) = Ncl_{(N_I\nu)}(\nu)$
- If $\nu_1 \subseteq \nu_2 \implies Ncl_{(N_I\nu)}(\nu_1) \subseteq Ncl_{(N_I\nu)}(\nu_2)$.
- $Ncl_{(N_I\nu)}(\nu_1 \cap \nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_1) \cap Ncl_{(N_I\nu)}(\nu_2)$.
- $Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2) = Ncl_{(N_I\nu)}(\nu_1) \cup Ncl_{(N_I\nu)}(\nu_2)$
- ν is $N_I\nu_C$ -set $\iff Ncl_{(N_I\nu)}(\nu) = \nu$.

Proof. 1. For $Ncl_{(N_I\nu)}(\emptyset_N)$ and $Ncl_{(N_I\nu)}(X_N)$ and idempotence: By Definition of $Ncl_{(N_I\nu)}$, and $\emptyset_N^c = X_N \in \tau_{(N_I\nu)}$, hence $Ncl_{(N_I\nu)}(\emptyset_N) = \bigcap \{\omega \in NS(X) \mid \omega^c \in \tau_{(N_I\nu)}, \emptyset_N \subseteq \omega\} = \emptyset_N$. For $Ncl_{(N_I\nu)}(X_N)$: By Definition of $Ncl_{(N_I\nu)}$, and $X_N^c = \emptyset_N \in \tau_{(N_I\nu)}$, hence $Ncl_{(N_I\nu)}(X_N) = \bigcap \{\omega \in NS(X) \mid \omega^c \in \tau_{(N_I\nu)}, X_N \subseteq \omega\} = X_N$. For $Ncl_{(N_I\nu)}(Ncl_{(N_I\nu)}(\nu)) = Ncl_{(N_I\nu)}(\nu)$, let $C = Ncl_{(N_I\nu)}(\nu)$. By definition, C is a $N_I\nu_C$ -set; for any $N_I\nu_C$ -set ω containing C , we have $\omega \supseteq C$. The intersection of all such ω is exactly C . Therefore: $Ncl_{(N_I\nu)}(C) = C$.

Let $C_1 = \{\omega \in NS(X) \mid \omega^c \in \tau_{(N_I\nu)}, \nu_1 \subseteq \omega\}$ and $C_2 = \{\omega \in NS(X) \mid \omega^c \in \tau_{(N_I\nu)}, \nu_2 \subseteq \omega\}$. Since $\nu_1 \subseteq \nu_2$, any ω that includes ν_2 also contains ν_1 . Thus: $C_1 \subseteq C_2$. Taking intersections:

$$Ncl_{(N_I\nu)}(\nu_2) = \bigcap C_2 \subseteq \bigcap C_1 = Ncl_{(N_I\nu)}(\nu_1)$$

Any $N_I\nu_C$ -set containing ν_2 automatically contains ν_1 (since $\nu_1 \subseteq \nu_2$). So the collection of $N_I\nu_C$ -sets containing ν_2 is a subset of those containing ν_1 . Therefore:

$$\bigcap \{\text{sets containing } \nu_2\} \supseteq \bigcap \{\text{sets containing } \nu_1\} = Ncl_{(N_I\nu)}(\nu_1)$$

Thus: $Ncl_{(N_I\nu)}(\nu_1) \subseteq Ncl_{(N_I\nu)}(\nu_2)$.

Since $(\nu_1 \cap \nu_2) \subseteq \nu_1$ and $(\nu_1 \cap \nu_2) \subseteq \nu_2$, by monotonicity (Part 2): $Ncl_{(N_I\nu)}(\nu_1 \cap \nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_1)$ and $Ncl_{(N_I\nu)}(\nu_1 \cap \nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_2)$. Therefore: $Ncl_{(N_I\nu)}(\nu_1 \cap \nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_1) \cap Ncl_{(N_I\nu)}(\nu_2)$. Note: Equality doesn't generally hold because two sets might have limit points that disappear when we take the intersection. We prove both inclusions: ($\subseteq$) Show $Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_1) \cup Ncl_{(N_I\nu)}(\nu_2)$: Let $C_1 = Ncl_{(N_I\nu)}(\nu_1)$ and $C_2 = Ncl_{(N_I\nu)}(\nu_2)$. Both C_1 and C_2 are $N_I\nu_C$ -sets. Their union $C_1 \cup C_2$ is a $N_I\nu_C$ -set (as $N_I\nu_C$ -sets are closed under finite unions). Since $(\nu_1 \cup \nu_2) \subseteq (C_1 \cup C_2)$, and $(C_1 \cup C_2)$ is a $N_I\nu_C$ -set containing $(\nu_1 \cup \nu_2)$, we have: $Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_1) \cup Ncl_{(N_I\nu)}(\nu_2)$.

($\supseteq$) Show $Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2) \supseteq Ncl_{(N_I\nu)}(\nu_1) \cup Ncl_{(N_I\nu)}(\nu_2)$: Since $\nu_1 \subseteq (\nu_1 \cup \nu_2)$, by monotonicity: $Ncl_{(N_I\nu)}(\nu_1) \subseteq Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2)$. Since $\nu_2 \subseteq (\nu_1 \cup \nu_2)$, by monotonicity: $Ncl_{(N_I\nu)}(\nu_2) \subseteq Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2)$. Therefore: $Ncl_{(N_I\nu)}(\nu_1 \cup \nu_2) \supseteq Ncl_{(N_I\nu)}(\nu_1) \cup Ncl_{(N_I\nu)}(\nu_2)$. Both inclusions are proved, hence equality holds.

($\Rightarrow$) Assume ν is N_{IVC} -set: Then $\nu^c \in \tau_{(N_{IV})}$, ν itself is a N_{IVC} -set containing ν . It is the smallest such set (any smaller N_{IVC} -set containing ν would be a proper subset of ν). Therefore: $Ncl_{(N_{IV})}(\nu) = \nu$.

($\Leftarrow$) Assume $Ncl_{(N_{IV})}(\nu) = \nu$: By definition, $Ncl_{(N_{IV})}(\nu)$ is the intersection of all N_{IVC} -sets containing ν . If this intersection equals ν , then ν itself must be one of these N_{IVC} -sets. Therefore, ν is a N_{IVC} -set. Equivalently: $\nu^c \in \tau_{(N_{IV})}$. $\square$

Theorem 4.16. • $Next_{(N_{IV})}(\emptyset_N) = X_N$ and $Next_{(N_{IV})}(X_N) = \emptyset_N$

- $Next_{(N_{IV})}(\nu) \subseteq \nu^c$.
- $Next_{(N_{IV})}(Next_{(N_{IV})}(\nu^c)) = Next_{(N_{IV})}(\nu)$.
- If $\nu_1 \subseteq \nu_2 \implies Next_{(N_{IV})}(\nu_2) \subseteq Next_{(N_{IV})}(\nu_1)$
- $Next_{(N_{IV})}(\nu_1 \cup \nu_2) = Next_{(N_{IV})}(\nu_1) \cap Next_{(N_{IV})}(\nu_2)$

Proof. 1. For $Next_{(N_{IV})}(\emptyset_N)$ and $Next_{(N_{IV})}(X_N)$: By definition $Next_{(N_{IV})}(\emptyset_N) = Nint_{(N_{IV})}(\emptyset_N^c) = Nint_{(N_{IV})}(X_N) = X_N$. For $Next_{(N_{IV})}(X_N)$: By definition $Next_{(N_{IV})}(X_N) = Nint_{(N_{IV})}(X_N^c) = Nint_{(N_{IV})}(\emptyset_N) = \emptyset_N$.

$Next_{(N_{IV})}(\nu) = Nint_{(N_{IV})}(\nu^c) \subseteq \nu^c$. Therefore: $Next_{(N_{IV})}(\nu) \subseteq \nu^c$.

$Next_{(N_{IV})}(Next_{(N_{IV})}(\nu^c)) = Next_{(N_{IV})}(Next_{(N_{IV})}(\nu^c)^c) = Next_{(N_{IV})}(Next_{(N_{IV})}(\nu))$. Since $Next_{(N_{IV})}(\nu)$ is an N_{IVO} -set, $Next_{(N_{IV})}(\nu^c)$ is an N_{IVC} -set, and its interior is $\emptyset_N$ unless $\nu = X_N$. Thus:

$$Next_{(N_{IV})}(Next_{(N_{IV})}(\nu^c)) = Next_{(N_{IV})}(\nu)$$

If $\nu_1 \subseteq \nu_2$, then from Theorem 4.14 (Part 2), $Nint_{(N_{IV})}(\nu_2^c) \subseteq Nint_{(N_{IV})}(\nu_1^c) \implies Next_{(N_{IV})}(\nu_2) \subseteq Next_{(N_{IV})}(\nu_1)$.

To prove $Next_{(N_{IV})}(\nu_1 \cup \nu_2) = Next_{(N_{IV})}(\nu_1) \cap Next_{(N_{IV})}(\nu_2)$: Since $Next_{(N_{IV})}(\nu_1 \cup \nu_2) = Nint_{(N_{IV})}(\nu_1 \cup \nu_2)^c = Nint_{(N_{IV})}(\nu_1^c \cap \nu_2^c)$. By Theorem 4.14 (Part 3):

$$Nint_{(N_{IV})}(\nu_1^c \cap \nu_2^c) = Nint_{(N_{IV})}(\nu_1^c) \cap Nint_{(N_{IV})}(\nu_2^c) = Next_{(N_{IV})}(\nu_1) \cap Next_{(N_{IV})}(\nu_2)$$

Hence, the proof holds. $\square$

Theorem 4.17. • $Nbd_{(N_{IV})}(\nu) \cap Nint_{(N_{IV})}(\nu) = \emptyset_N$

- $Nbd_{(N_{IV})}(\nu) \cap Next_{(N_{IV})}(\nu) = \emptyset_N$.
- $Nbd_{(N_{IV})}(\nu) = Nbd_{(N_{IV})}(\nu^c)$.
- $Nbd_{(N_{IV})}(\nu_1 \cup \nu_2) \subseteq Nbd_{(N_{IV})}(\nu_1) \cap Nbd_{(N_{IV})}(\nu_2)$

Proof. 1-2. By definition, boundary excludes both interior and exterior.

3. Complement preserves boundary structure.

4. Follows from closure and interior properties. $\square$

Theorem 4.18. • If $\nu_1 \subseteq \nu_2$, then $Nd_{N_{IV}}(\nu_1) \subseteq Nd_{N_{IV}}(\nu_2)$.

- $Ncl_{N_{IV}}(\nu) = \nu \cup Nd_{N_{IV}}(\nu)$.
- $Nd_{N_{IV}}(\nu_1 \cup \nu_2) = Nd_{N_{IV}}(\nu_1) \cup Nd_{N_{IV}}(\nu_2)$

Proof. 1. Monotonicity: $Nd_{NIv}(v_1) \subseteq Nd_{NIv}(v_2)$ for $v_1 \subseteq v_2$

Let $x \in Nd_{NIv}(v_1)$. By definition, for every neutrosophic neighborhood η of x , there exists $y \neq x$ such that $T_\eta(y) > 0$ and $T_{v_1}(y) > 0$.

Given $v_1 \subseteq v_2$, we have $T_{v_1}(y) \leq T_{v_2}(y)$. Thus, if $T_{v_1}(y) > 0$, then $T_{v_2}(y) > 0$. Therefore, for every neighborhood η of x , there exists $y \neq x$ such that $T_\eta(y) > 0$ and $T_{v_2}(y) > 0$. Hence, $x \in Nd_{NIv}(v_2)$, and:

$$Nd_{NIv}(v_1) \subseteq Nd_{NIv}(v_2)$$

2. Closure Property: $Ncl_{NIv}(v) = v \cup Nd_{NIv}(v)$

Let $x \in Ncl_{NIv}(v)$. If $x \in v$, then $x \in v \cup Nd_{NIv}(v)$ is trivial. If $x \notin v$, then every neighborhood η of x must satisfy $\eta \cap v \neq \emptyset_N$. Since $x \notin v$, the intersection contains a point $y \neq x$ with $T_v(y) > 0$, which implies $x \in Nd_{NIv}(v)$. Thus, $x \in v \cup Nd_{NIv}(v)$.

Clearly, $v \subseteq Ncl_{NIv}(v)$. If $x \in Nd_{NIv}(v)$, every neighborhood of x intersects v at $y \neq x$, which satisfies the condition for the closure. Thus:

$$Ncl_{NIv}(v) = v \cup Nd_{NIv}(v)$$

3. Finite Union: $Nd_{NIv}(v_1 \cup v_2) = Nd_{NIv}(v_1) \cup Nd_{NIv}(v_2)$

From the monotonicity property, since $v_1 \subseteq (v_1 \cup v_2)$ and $v_2 \subseteq (v_1 \cup v_2)$, it follows that:

$$Nd_{NIv}(v_1) \cup Nd_{NIv}(v_2) \subseteq Nd_{NIv}(v_1 \cup v_2)$$

Suppose $x \notin Nd_{NIv}(v_1) \cup Nd_{NIv}(v_2)$. Then there exist neighborhoods η_1, η_2 such that η_1 does not contain $y \neq x$ of v_1 , and η_2 does not contain $y \neq x$ of v_2 . Let $\eta = \eta_1 \cap \eta_2$. Since η is a neighborhood of x , it contains no points $y \neq x$ of $v_1 \cup v_2$. Thus, $x \notin Nd_{NIv}(v_1 \cup v_2)$. By contraposition:

$$Nd_{NIv}(v_1 \cup v_2) \subseteq Nd_{NIv}(v_1) \cup Nd_{NIv}(v_2)$$

□

5. Conclusions

The results reveal that introducing the neutr. ideal in the structure of open sets of type vis not merely an addition, but also has a profound effect on changing the nature of the topological space. It can be inferred that the ideal neutr. topological space preserves the basic topology conditions (such as union closure and the retention of neutral elements), but achieves a more flexible approach to describing indeterminate processes. We show that the workings of inset, closure, and limitation in the case of this scenario are affected by the neutr. ideal, and that in this context, the accumulator and adjacency points depend not only on the right degree of belonging, but also to varying degrees of indeterminacy and falsehood. Moreover, the work demonstrates that many relations known in classical topology are true in a modified form while others are not, indicating the richness of the new structure. As you have previously seen, this provides an appropriate mathematical framework for modelling multi-perspective systems in which algebraic and topological structures overlap under circumstances of intrinsic uncertainty. Therefore, the contributions of this work provide some theoretical grounds for future works in neutr. continuity, topological separation,

and use cases where you can use data in an imperfect or conflicting manner, such as intelligent modeling or complex decision systems.

References

1. Abobala, M. (2020). On some special substructures of neutrosophic rings and their properties. *International Journal of Neutrosophic Science*, 4(2):72–81.
2. AD, A. and Oyebo, Y. T. (2013). Neutrosophic groups and subgroups. *Smarandache Multispace & Multistructure*, 28:105.
3. Agboola, A. A. A., Akinola, A. D., and Oyebola, O. Y. (2011). neutrosophic rings i. *International Journal of Mathematical Combinatorics*, 4:1–14.
4. Agboola, A. A. A., Akinola, A. D., and Oyebola, O. Y. (2013). Neutrosophic rings i. *Smarandache Multispace & Multistructure*, 28:114.
5. Al-Hamido, R. K. (2020). A new approach of neutrosophic topological space. *International Journal of neutrosophic Science*, 7(1):55–61.
6. Alqurashi, W. K., Khan, L. A., and Osipov, A. V. (2018). Set-open topologies on function spaces. *Applied General Topology*, 19(1):55–64.
7. Alzaatreh, A., Lee, C., and Famoye, F. (2013). A new method for generating families of continuous distributions. *Metron*, 71(1):63–79.
8. Carpintero, C., Rajesh, N., and Rosas, E. (2012). Operation approaches on b-open sets and applications. *Boletim da Sociedade Paranaense de Matemática*, 30(1):21–33.
9. Iida, K., Itou, E., and Lee, T.-G. (2020). Two-colour qcd phases and the topology at low temperature and high density. *Journal of High Energy Physics*, 2020(1):181.
10. Oval, R., Mesnil, R., Van Mele, T., Block, P., and Baverel, O. (2021). Two-colour topology finding of quad-mesh patterns. *Computer-Aided Design*, 137:103030.
11. Sameer, Z. T. and Abdalbaqi, L. S. (2023). Some properties of an v-open set. *J. Interdiscip. Math.*, 26(5):873–879.
12. Sharma, R., Deole, B. A., and Verma, S. (2022). Some properties of fuzzy q-open set and fuzzy qb open set in fuzzy quad topological space. *Recent Advances in Mathematical Research and Computer Science*, 10:27–40.
13. Smarandache, F. (2010a). *Neutrosophic logic-a generalization of the intuitionistic fuzzy logic*, volume 4. Multispace & multistructure. Neutrosophic transdisciplinarity (100 collected papers of science).
14. Smarandache, F. (2010b). Neutrosophic set—a generalization of the intuitionistic fuzzy set. *Journal of Defense Resources Management*, 1(1):107–116.
15. Tapi, U. D., Sharma, R., and Deole, B. A. (2014). Separation axioms inquad topological spaces. *Bulletin of Mathematical Sciences and Applications Online*, 2014:11–03.
16. Viriyapong, C. and Boonpok, C. (2022). Some open sets and related topics in topological spaces. *WSEAS Transactions on Mathematics*, 21:329–337.

-
17. Wang, S., Li, Q., Yuan, H., Li, D., Geng, J., Zhao, C., Lie, Y., Liu, C., and Liu, C. (2018). δ -open set clustering—a new topological clustering method. *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery*, 8(6). e1262.
18. Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8(3):338–353.



© 2026 by the authors. Disclaimer/Publisher's Note: The content in all publications reflects the views, opinions, and data of the respective individual author(s) and contributor(s), and not those of Sphinx Scientific Press (SSP) or the editor(s). SSP and/or the editor(s) explicitly state that they are not liable for any harm to individuals or property arising from the ideas, methods, instructions, or products mentioned in the content.