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Research article

Study Relative Loss Values for Entropy Functions of Lindley and Truncated Lindley Distributions

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ABSTRACT

Entropy is a measure of uncertainty in probability distributions when studying different phenomena. Numerical comparisons of the relative loss in entropy are calculated to determine which entropy measure has better advantages over the other. In this paper, five entropy measures are derived for the Lindley and truncated Lindley distributions to calculate the relative loss of entropy of the two distributions when the underlying probability distribution is the truncated Lindley distribution instead of the Lindley distribution, and to determine the best uncertainty measure based on the relative loss results.

1. Introduction

Accurate measurements of a particular phenomenon in daily life are important for measuring potential gains in both money and time. Uncertainty is implicit in all measurements and on different scales, which in turn requires knowing how to calculate uncertainty to better understand the accuracy of the phenomenon measurements. In fact, entropy is a measure of disorder and chaos in a given system and plays a fundamental role in determining the direction of changes occurring in that system, which has prompted many to be interested in studying the types of entropy for many phenomena based on their statistical distributions, including Jasim et al. [5] proposed the novel odd generalized exponential-G family, statistical properties and applications, Al-Habib et al. [4] introduced the statistical properties and application for [0, 1] truncated Nadarajah-Haghighi exponential distribution, and Alotaibi et al. [6] developed the half logistic inverted Nadarajah–Haghighi distribution under ranked set sampling with applications and derived various kinds of entropy, such as Tsallis entropy, the Rényi entropy, Havrda and Charvat entropy and Arimoto entropy. The relative loss of entropy describes the extent to which a system is able to maintain a degree of order and organization despite the increasing chaos in the surrounding environment.

This is an important concept in understanding the physical, chemical, and other systems processes. Statisticians and researchers have been interested in comparing entropy measures based on the relative loss of these measures. Basit et al. [2] calculated the relative loss function for eight entropy measures on biased, truncated, and normal exponential distributions. Dey et al. [1] used the truncated Rayleigh distribution instead of the original Rayleigh distribution to calculate the relative loss function for seven entropy measures. Ahsan and Aslam [3] calculated the relative loss function for nine entropy measures, and used the truncated Gompertz distribution instead of the original Gompertz distribution to achieve the objective of the study. In this paper, some uncertainty measures for probability distributions will be discussed to show the causes of uncertainty and we will provide steps to calculate uncertainty in working measurements.

Our paper is structured as follows: Section 2 presents the Lindley distribution and the proposed truncated Lindley distribution ([0, k]). Section 3 presents the extensions of probability density functions (PDFs) for both the Lindley distribution and the truncated Lindley distribution ([0, k]). Section 4 examines the entropy measures of the Lindley distribution, while Section 5 studies the entropy measures of the proposed truncated Lindley distribution ([0,k]). Section 6 discusses the relative loss of entropy between the two distributions. Section 7 provides a comprehensive numerical analysis to illustrate and compare the entropy measures under different coefficient settings. Finally, Section 8 concludes the paper by summarizing the main findings and highlighting potential directions for future research.

2. Lindley and $[0, k]$ truncated Lindley distributions

Lindley distribution is one of the most important statistical distributions introduced by Lindley [12], where the random ($Z\sim$ Lindley distribution) which has the following Cdf and Pdf respectively:

$$F(z,a) = 1 - \left(1 + \frac{az}{1+a}\right) e^{-az} \qquad z > 0, \qquad a > 0, \tag{1.1}$$

$$f(z,a) = \frac{a^2}{1+a}(1+z)e^{-az} \qquad z > 0, \qquad a > 0. \tag{1.2}$$

The truncated process will be performed on this distribution within the interval $[\mathbf{0}, \boldsymbol{k}]$ in order to obtain the truncated Lindley distribution with the aim of studying the relative loss differences between the two distributions for several entropy functions if the truncated distribution is the basis for the operation.

In this paper a new truncated distribution named ($T \sim [\mathbf{0}, \boldsymbol{k}]$ truncated Lindley distribution) is proposed has the following Cdf and Pdf respectively for studying the relative loss function of various entropy kinds:

$$F(t,a) = \frac{1 - \left(1 + \frac{at}{1+a}\right) e^{-at}}{1 - \left(1 + \frac{ak}{1+a}\right) e^{-ak}} \qquad t \in [0, k], \qquad a > 0, \tag{1.3}$$

$$f(t,a) = \frac{\frac{a^2}{1+a}(1+t)e^{-at}}{1 - \left(1 + \frac{ak}{1+a}\right) e^{-ak}} \qquad t \in [0, k], \qquad a > 0. \tag{1.4}$$

The survival of $[\mathbf{0}, \boldsymbol{k}]$ truncated Lindley distribution is given as follows:

$$S(t,a) = 1 - \frac{1 - \left(1 + \frac{at}{1+a}\right) e^{-at}}{1 - \left(1 + \frac{ak}{1+a}\right) e^{-ak}} \qquad t \in [0, k], \qquad a > 0 \tag{1.5}$$

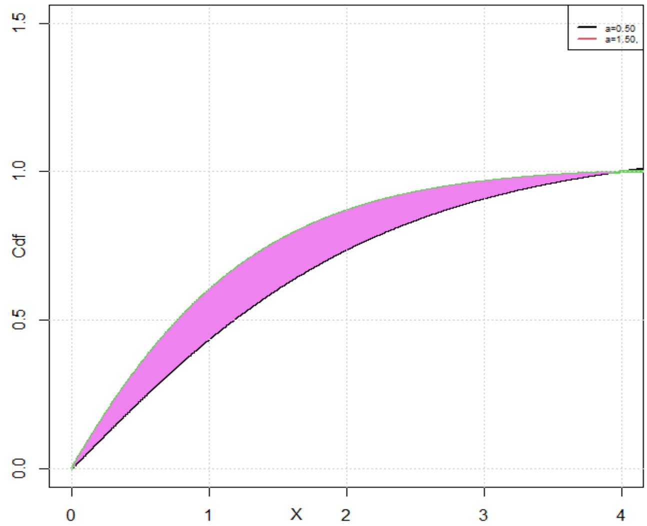
and the hazard of $[\mathbf{0}, \boldsymbol{k}]$ truncated Lindley distribution is given as follows:

$$h(t,a) = \frac{1 - \dfrac{1 - \left(1 + \frac{at}{1+a}\right) e^{-at}}{1 - \left(1 + \frac{ak}{1+a}\right) e^{-ak}}}{\dfrac{\frac{a^2}{1+a}(1+t)e^{-tz}}{\frac{a^2}{1+a}(1+a)e^{-ak}}} \qquad t \in [0, k], \qquad a > 0.$$

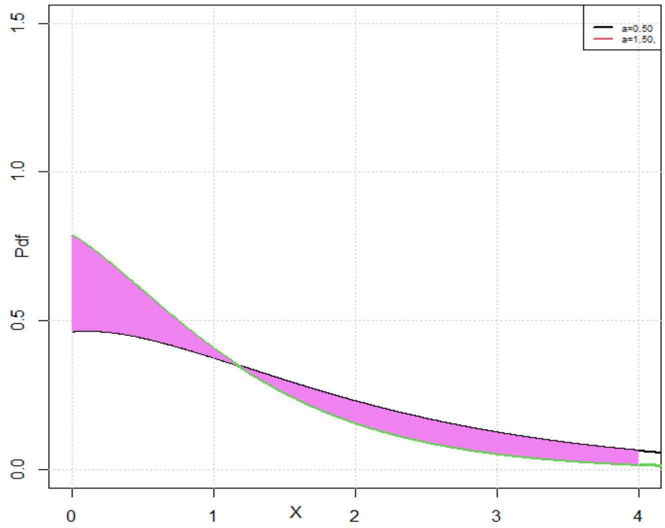
This can be Simplify to obtained

$$h(t,a) = \frac{\left(-2\left(1+\frac{ak}{1+a}\right)e^{-ak} - \left(1+\frac{at}{1+a}\right)e^{-at}\right)\left[\frac{a^2}{1+a}(1+a)e^{-ak}\right]}{\frac{a^2}{1+a}(1+t)e^{-tz}} \qquad t \in [0,k], \qquad a > 0. \tag{1.6}$$

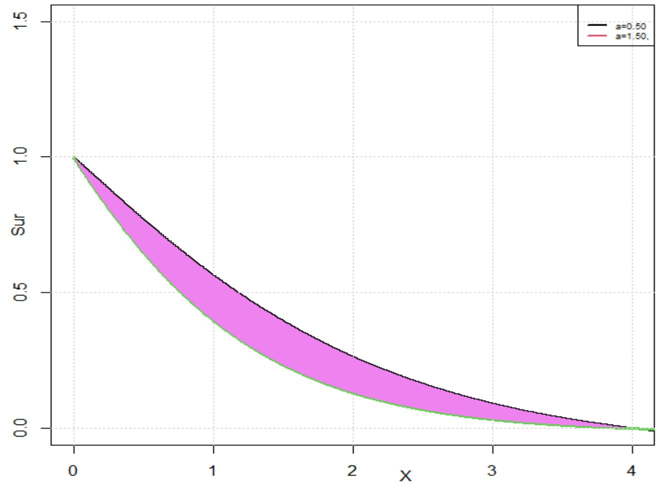
The following graphs in Figure 1 illustrate the behavior of the above functions at different values of the parameters and depending on the value of $k = 4$.



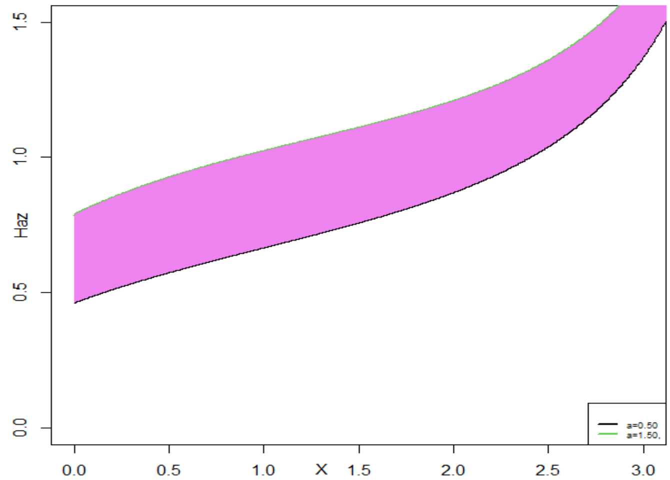
(a) CDF of $[\mathbf{0}, \mathbf{4}]$ truncated Lindley distribution.



(b) PDF of $[\mathbf{0}, \mathbf{4}]$ truncated Lindley distribution.



(c) Survival function of $[\mathbf{0}, \mathbf{4}]$ truncated Lindley distribution.



(d) Hazard function $[\mathbf{0}, \mathbf{4}]$ truncated Lindley distribution.

Figure 1. Graphical representation of proposed model CDF, PDF, survival function, and hazard function of the proposed model.

3. Expansion Pdf for Lindley and $[\mathbf{0}, \boldsymbol{k}]$ truncated Lindley distributions

Some types of entropy depend on the expanded probability density function raised to the power δ, which represents the index of entropy. In this section, the Lindley probability density function with power which represents the entropy index as follows:

$$\left(f(z,a)\right)^{\delta} = \left(\frac{a^2}{1+a}(1+z)e^{-az}\right)^{\delta} = \left(\frac{a^2}{1+a}\right)^{\delta}(1+z)^{\delta}\, e^{-\delta a z}$$

$$(1+z)^{\delta}=\sum_{i=0}^{\infty}\binom{\delta}{i}z^{i}$$

Then

$$\left(f(z,a)\right)^{\delta}=\left(\frac{a^2}{1+a}\right)^{\delta}\sum_{i=0}^{\infty}\binom{\delta}{i}z^{i}\,e^{-\delta az} \qquad (2.1)$$

Now the expansion Pdf for $[0,k]$ truncated Lindley distribution with power c is given by:
According to Eq. (2.1) as follows:

$$\left(f(t,a)\right)^{\delta}=\left(\frac{\frac{a^2}{1+a}(1+t)e^{-at}}{1-\left(1+\frac{ak}{1+a}\right)e^{-ak}}\right)^{\delta}=\left(\frac{\frac{a^2}{1+a}}{1-\left(1+\frac{ak}{1+a}\right)e^{-ak}}\right)^{\delta}\sum_{j=0}^{\infty}\binom{\delta}{j}t^{j}\,e^{-\delta at}$$

Let $\Psi_j=\left(\frac{\frac{a^2}{1+a}}{1-\left(1+\frac{ak}{1+a}\right)e^{-ak}}\right)^{\delta}\sum_{j=0}^{\infty}\binom{\delta}{j}$.

Then

$$\left(f(t,a)\right)^{\delta}=\Psi_j\,t^{j}e^{-\delta at}. \qquad (2.2)$$

Now for each one of Renyi, HC, Ts, AR entropy assumes that $\delta=c$ and for MH entropy let $\delta=2-c$

4. Entropy measures of Lindley distribution

In this part, various entropy measures for Lindley distribution will be derived.

Rényi [10] introduced Renyi entropy and it is calculated for Lindley distribution as follows:

$I_R(\delta)=\frac{1}{1-\delta}log\left[\left(\frac{a^2}{1+a}\right)^{\delta}\sum_{i=0}^{\infty}\binom{\delta}{i}\int_0^{\infty}z^{i}\,e^{-\delta az}\,dz\right],\qquad \delta>0\,,\delta\neq 1.$

Let $u=\delta az\ \rightarrow z=\frac{1}{\delta a}u\ \rightarrow dz=\frac{1}{\delta a}du.$

So that

$\left(\frac{1}{\delta a}\right)^{i+1}\int_0^{\infty}(u)^{\,i}\,e^{-u}\,du=\left(\frac{1}{\delta a}\right)^{i+1}\Gamma(i+1).$

So that

$$I_R(\delta)=\frac{1}{1-\delta}\log\left[\left(\frac{a^2}{1+a}\right)^{\delta}\sum_{i=0}^{\infty}\binom{\delta}{i}\left(\frac{1}{\delta a}\right)^{i+1}\Gamma(i+1)\right]. \qquad (3.1)$$

Arimoto [8] proposed Armito entropy and it is calculated for Lindley distribution as follows:

$$A_{\delta}=\frac{\delta}{1-\delta}\left(\left(\varphi_i\left(\frac{1}{\delta a}\right)^{\delta+1}\Gamma(\delta+1)\right)^{\frac{1}{\delta}}-1\right),\qquad \delta>0\,,\qquad \delta\neq 1 \qquad (3.2)$$

Havrda and Charvát [9] developed Havrda and Charvat entropy and it is calculated for Lindley distribution as follows:

$$HC_{\delta}=\frac{1}{2^{1-\delta}-1}\left(\left(\varphi_i\left(\frac{1}{\delta a}\right)^{\delta+1}\Gamma(\delta+1)\right)^{\frac{1}{\delta}}-1\right)\qquad \delta>0\,,\delta\neq 1. \qquad (3.3)$$

Tsallis [7] defined Tsallis entropy and it is calculated for Lindley distribution as follows:

$$T_{\delta}=\frac{1}{\delta-1}\left(1-\left(\varphi_i\left(\frac{1}{\delta a}\right)^{\delta+1}\Gamma(\delta+1)\right)\right)\qquad \delta>0,\qquad \delta\neq 1. \qquad (3.4)$$

Paul and Thomas [11] introduced Mathai–Haubold entropy and it is calculated for Lindley distribution as follows:

$$MH_{\delta} = \left(\varphi_i \left(\frac{1}{(2-\delta)a}\right)^{2-\delta} \Gamma(2-\delta)\right). \tag{3.5}$$

5. Entropy measures of $[0, \boldsymbol{k}]$ truncated Lindley distribution

In this part, various entropy measures for $[0, k]$ truncated Lindley distribution will be derived as follows:
Renyi entropy of $[0, k]$ truncated Lindley distribution is given by

$I_R(\delta) = \frac{1}{1-\delta} log\left[\Psi_j \int_0^k t^j e^{-\delta a t} dt\right]. \quad \delta > 0 , \delta \neq 1.$
$Let\ \ w = \delta a t \ \to t = \frac{1}{\delta a} w \ \to dt = \frac{1}{\delta a} dw$
For $t = 0 \ \to w = 0$ and if $t = k \ \to w = \delta a k$
So that

$$\int_0^k t^j e^{-a\delta t} dt = \int_0^{ak\delta} \left(\frac{1}{a\delta} w\right)^j e^{-w} \frac{1}{a\delta} dw = \left(\frac{1}{a\delta}\right)^{j+1} \int_0^{\delta ak} w^j e^{-w} dw = \left(\frac{1}{\delta a}\right)^{j+1} \gamma(j+1, \delta ak),$$

$$I_R(\delta) = \frac{1}{1-\delta} log\left[\Psi_j \left(\frac{1}{a\delta}\right)^{j+1} \gamma(j+1, \delta ak)\right]. \tag{4.1}$$

Armito entropy of $[0, k]$ truncated Lindley distribution is given by

$$A_{\delta} = \frac{\delta}{1-\delta}\left(\Psi_j \left(\left(\frac{1}{a\delta}\right)^{j+1} \gamma(j+1, \delta ak)\right)^{\frac{1}{\delta}} - 1\right) \ , \ \delta > 0 , \quad \delta \neq 1. \tag{4.2}$$

Havrda and Charvat entropy of $[0, k]$ truncated Lindley distribution is given by:

$$HC_{\delta} = \frac{1}{2^{1-\delta} - 1}\left(\Psi_j \left(\left(\frac{1}{a\delta}\right)^{j+1} \gamma(j+1, \delta ak)\right)^{\frac{1}{\delta}} - 1\right) \qquad \delta > 0 , \delta \neq 1. \tag{4.3}$$

Tsallis entropy of $[0, k]$ truncated Lindley distribution is given by:

$$T_{\delta} = \frac{1}{\delta - 1}\left(1 - \Psi_j \left(\left(\frac{1}{a\delta}\right)^{j+1} \gamma(j+1, \delta ak)\right)^{\frac{1}{\delta}} - 1\right) \qquad \delta > 0, \quad \delta \neq 1. \tag{4.4}$$

Mathai–Haubold entropy $[0, k]$ truncated Lindley distribution is given by:

$$MH_C = \left(\frac{a^2}{1+a}(1+a)\right)^{2-c} \left(\frac{1}{(2-c)a}\right)^{2-c} \Gamma(c+1). \tag{4.5}$$

6. Relative loss of entropy

In this part will introduce the relative loss function relative loss of entropy by using *Z* instead of *T* based on different measures of entropy as follows:
If *M*(*X*) and *M*(*Y*) are two corresponding entropies, then the relative loss of entropy in using *T* instead of *Z* is defined as follows:
$RL_{M(Z)} = \frac{M(Z) - M(T)}{M(Z)}$
Relative loss of Renyi entropy in using *T* instead of *Z* is given by:

$$RL_{RM(Z)} = \frac{\frac{1}{1-\delta}\, log\left[\varphi_i \left(\frac{1}{a\delta}\right)^{i+1} \Gamma(i+1)\right] - \frac{1}{1-\delta} log\left[\Psi_j \left(\frac{1}{\delta a}\right)^{j+1} \gamma(j+1,\delta ak)\right]}{\frac{1}{1-\delta}\, log\left[\varphi_i \left(\frac{1}{a\delta}\right)^{i+1} \Gamma(i+1)\right]}.$$

Relative loss of Armito entropy in using T instead of Z is given by:

$$RL_{ARM(Z)} = \frac{\frac{\delta}{1-\delta}\left(\left(\varphi_i \left(\frac{1}{a\delta}\right)^{c+1} \Gamma(\delta+1)\right)^{\frac{1}{\delta}} - 1\right) - \frac{\delta}{1-\delta}\left(\Psi_j \left(\left(\frac{1}{a\delta}\right)^{j+1} \gamma(j+1,\delta ak)\right)^{\frac{1}{\delta}} - 1\right)}{\frac{\delta}{1-\delta}\left(\left(\varphi_i \left(\frac{1}{a\delta}\right)^{\delta+1} \Gamma(\delta+1)\right)^{\frac{1}{\delta}} - 1\right)}.$$

Relative loss of Havrda and Charvat entropy in using T instead of Z is given by:

$$RL_{HCM(Z)} = \frac{\frac{1}{2^{1-\delta}-1}\left(\left(\varphi_i \left(\frac{1}{a\delta}\right) \delta^{+1}\, \Gamma(\delta+1)\right)^{\frac{1}{\delta}} - 1\right) - \frac{1}{2^{1-\delta}-1}\left(\Psi_j \left(\left(\frac{1}{a\delta}\right)^{j+1} \gamma(j+1,\delta ak)\right)^{\frac{1}{\delta}} - 1\right)}{\frac{1}{2^{1-\delta}-1}\left(\left(\varphi_i \left(\frac{1}{a\delta}\right)^{\delta+1} \Gamma(\delta+1)\right)^{\frac{1}{\delta}} - 1\right)}.$$

Relative loss of Tsallis entropy in using T instead of Z is given by:

$$RL_{TsM(Z)} = \frac{\frac{1}{\delta-1}\left(1 - \left(\varphi_i \left(\frac{1}{a\delta}\right)^{\delta+1} \Gamma(\delta+1)\right)\right) - \frac{1}{\delta-1}\left(1 - \Psi_j \left(\left(\frac{1}{\delta a}\right)^{j+1} \gamma(j+1,\delta ak)\right)^{\frac{1}{\delta}} - 1\right)}{\frac{1}{\delta-1}\left(1 - \left(\varphi_i \left(\frac{1}{\delta a}\right)^{\delta+1} \Gamma(\delta+1)\right)\right)}.$$

Relative loss of Mathai–Haubold entropy in using T instead of Z is given by:

$$RL_{M(Z)} = \frac{\left(\varphi_i \left(\frac{1}{(2-\delta)a}\right)^{2-\delta} \Gamma(2-\delta)\right) - \left(\frac{a^2}{1+a}(1+a)\right)^{2-\delta} \left(\frac{1}{(2-\delta)a}\right)^{2-\delta} \Gamma(\delta+1)}{\left(\varphi_i \left(\frac{1}{(2-\delta)a}\right)^{3-c} \Gamma(2-\delta)\right)}.$$

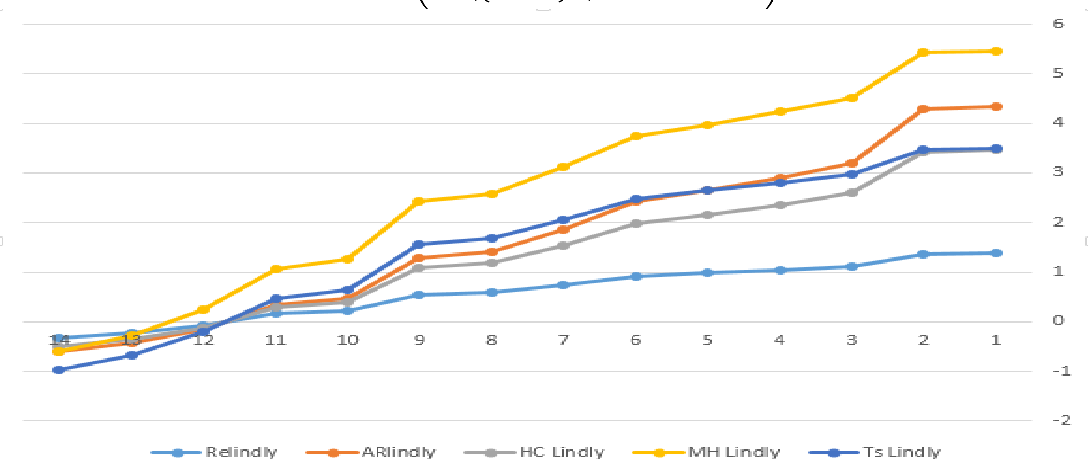


Figure 2. Entropies value for Table 1.

Table 1. Entropy values for Lindley distribution using value $c = 0.5$.

$\boldsymbol{a}$	$\boldsymbol{\delta}$	**Re Lindley**	**AR Lindley**	**HC Lindley**	**MH Lindley**	**Ts Lindley**
0.01	0.5	1.376344	2.960396	-0.86708	1.999875	1.980149
0.02	0.5	1.366492	2.921569	-0.85571	1.999482	1.96059
0.3	0.5	1.12393	2.076923	-0.60832	1.919041	1.508232
0.4	0.5	1.049822	1.857143	-0.54394	1.871215	1.380617
0.5	0.5	0.980829	1.666667	-0.48816	1.818556	1.265986
0.6	0.5	0.916291	1.5	-0.43934	1.762829	1.162278
0.9	0.5	0.744441	1.105263	-0.32372	1.587624	0.901905
1.2	0.5	0.597837	0.818182	-0.23964	1.411607	0.6968
1.3	0.5	0.553385	0.73913	-0.21649	1.353998	0.637522
2.2	0.5	0.223144	0.25	-0.07322	0.872649	0.236068
2.4	0.5	0.162519	0.176471	-0.05169	0.774981	0.169305
3.3	0.5	-0.07232	-0.06977	0.020434	0.371589	-0.07103
4	0.5	-0.22314	-0.2	0.058579	0.091889	-0.21115
4.5	0.5	-0.31845	-0.27273	0.07988	-0.09324	-0.29439

The appearance of negative values in the results of some entropy functions indicates that the uncertainty is very low.

Table 2. Entropy values for Lindley distribution using value $c = 1.3$.

a	δ	Re Lindley	AR Lindley	HC Lindley	MH Lindley	Ts Lindley
0.01	1.3	25.47875	4.321219	-0.2305	-2.58388	3.331736
0.02	1.3	21.82461	4.305181	-0.22964	-2.35122	3.328554
0.3	1.3	8.432648	3.714339	-0.19813	-0.88501	3.067749
0.4	1.3	7.219478	3.514354	-0.18746	-0.72529	2.951156
0.5	1.3	6.328348	3.327367	-0.17749	-0.61624	2.834025
0.6	1.3	5.635633	3.152989	-0.16818	-0.53978	2.718692
0.9	1.3	4.217837	2.696129	-0.14381	-0.42027	2.392866
1.2	1.3	3.318814	2.318661	-0.12368	-0.38377	2.101716
1.3	1.3	3.084544	2.206745	-0.11771	-0.38111	2.012042
2.2	1.3	1.709762	1.41277	-0.07536	-0.44169	1.337535
2.4	1.3	1.508408	1.273859	-0.06795	-0.46368	1.21326
3.3	1.3	0.827626	0.753392	-0.04019	-0.56793	0.732882
4	1.3	0.455209	0.432115	-0.02305	-0.64597	0.425494
4.5	1.3	0.240043	0.233516	-0.01246	-0.69814	0.231604

The negative values that appeared in the study tables indicate that the method or entropy function studied gave less loss than the reference, and therefore its performance is better.

7. Numerical analysis

In this section, we will discuss the numerical analysis of entropy values for selected values of parameters (a, k, c) according to the permissible limits, where tables of entropy values of the two distributions were built according to the parameters, supported by graphs for the purpose of facilitating understanding and conclusion, as in Tables 1, 2, 3, and 4. In addition to calculating the relative loss in Tables 5 and 6. For some of the parameters that were used previously, where they are displayed for all. A study entropy for Lindley (Z) with a comparison $[0, k]$ truncated Lindley (T) distributions.

One can be noted the following important points:

1. Through the values of the entropy functions of the Lindley distribution in Tables 1 and 2, the entropy measures showed a decreasing behavior when the value of the distribution coefficient increased, with respect to the stability of the entropy index, as shown in Figures 2 and 3.
2. The entropy behavior of the truncated distribution when increasing the value of the truncated while the distribution parameter and entropy index are constant is increasing and meets at one point so that the entropy functions take the same value when the value of the truncated is one and begins to increase again, separating from each other with the same increasing behavior until the end of the chosen truncated value, as in Table 3 and Figure 4. It is clear from the four states and what is also supported by Figure 5 that increasing the value of the truncated while the distribution parameter and entropy index are constant remains the same increasing behavior and meets at the value of one for the truncated to take an increasing convergent trend for the values of disorder within the interval [0,1].
3. It is clear from the Table 5 and Figure 6 that the relative loss values took a decreasing behavior for all entropy functions and the Havrda and Charvat and Armito functions, were the most relative loss compared to the rest of the functions, while all the functions had almost the same relative loss value when the amount of the truncation was one to return to their decreasing behavior after that value in another order as follows: Havrda and Charvat and Armito had the same value, and the behavior of the relative loss of the Renyi and Tsails functions converged to finally solve the relative loss value of the function Mathai–Haubold entropy. Also, Table 6 has a graphical representation in Figure 7.
4. The values of the relative loss function of the Renyi, Havrda and Charvat, Armito, and Tsails functions start decreasing to stabilize their behavior when the value of the cutoff is three, while the behavior of the Mathai–Haubold function is increasing from the beginning.

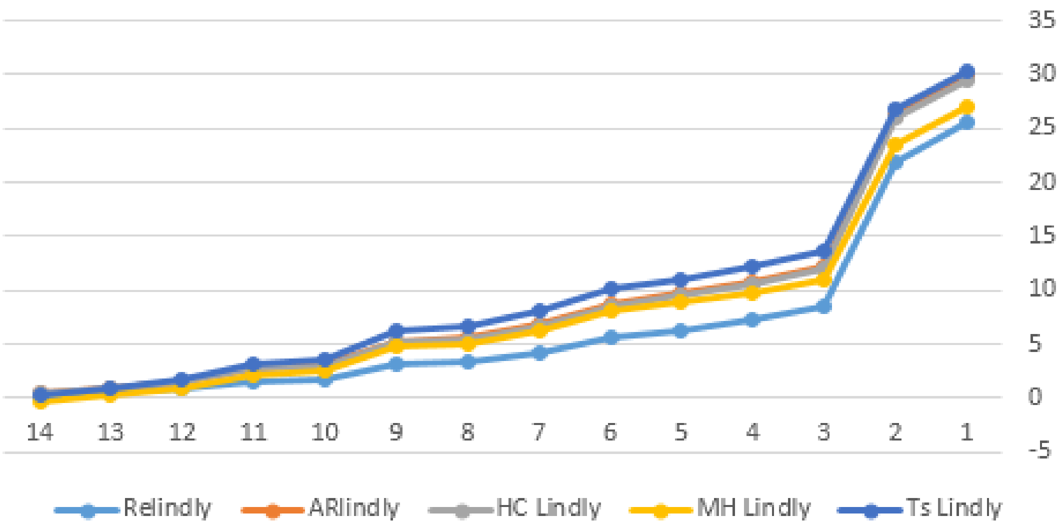


Figure 3. Entropies value for Table 2.

Table 3. Entropy values for$[0, k]$ truncated Lindley distribution using value $c = 0.5$.

a	δ	t	Re TLindley	AR TLindley	HC TLindley	MH TLindley	Ts TLindley
0.6	0.5	0.25	-1.38641	-0.75003	0.219678	-2.00066	-1.00006
0.6	0.5	0.5	-0.69338	-0.50011	0.14648	-0.8294	-0.58595
0.6	0.5	0.75	-0.28794	-0.25019	0.073279	-0.31027	-0.26817
0.6	0.5	1	-0.00021	-0.00021	0.00006	-0.00063	-0.00021
0.6	0.5	1.25	0.222966	0.249778	-0.07316	0.210673	0.235869
0.6	0.5	1.5	0.405239	0.499661	-0.14635	0.366456	0.449213
0.6	0.5	1.75	0.559184	0.749244	-0.21945	0.487172	0.64518
0.6	0.5	2	0.692285	0.998277	-0.29239	0.583989	0.827208
0.6	0.5	2.25	0.809358	1.246464	-0.36508	0.663597	0.997642
0.6	0.5	2.5	0.913679	1.493478	-0.43743	0.730284	1.15815
0.6	0.5	2.75	1.00758	1.738965	-0.50933	0.786947	1.309964
0.6	0.5	3	1.092781	1.982557	-0.58068	0.835629	1.454016
0.6	0.5	3.25	1.170585	2.223878	-0.65136	0.877818	1.591032
0.6	0.5	3.5	1.242005	2.462547	-0.72126	0.914632	1.721584

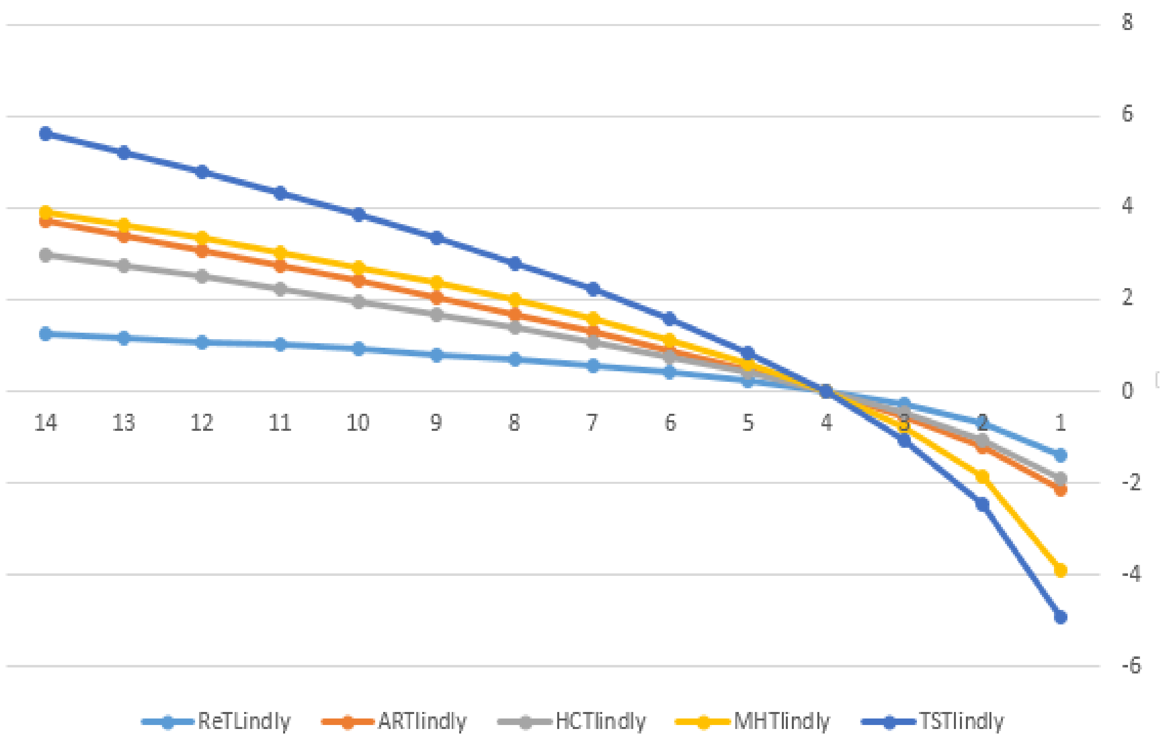


Figure 4: Entropies value for Table 3.

Table 4. Entropy values for $[0,k]$ truncated Lindley distribution using value $c = 1.3$

a	δ	t	Re TLindley	AR TLindley	HC TLindley	MH TLindley	Ts TLindley
2.2	1.3	0.25	-1.39205	-1.64164	0.087567	-1.1362	-1.72779
2.2	1.3	0.5	-0.71873	-0.78177	0.041701	-0.63711	-0.8021
2.2	1.3	0.75	-0.34893	-0.36337	0.019382	-0.30643	-0.36785
2.2	1.3	1	-0.11224	-0.1137	0.006065	-0.06236	-0.11415
2.2	1.3	1.25	0.046792	0.04654	-0.00248	0.124359	0.046465
2.2	1.3	1.5	0.155013	0.152273	-0.00812	0.268697	0.151464
2.2	1.3	1.75	0.228482	0.222563	-0.01187	0.380184	0.220828
2.2	1.3	2	0.277912	0.269188	-0.01436	0.465775	0.266642
2.2	1.3	2.25	0.310784	0.299901	-0.016	0.530932	0.296736
2.2	1.3	2.5	0.332381	0.319954	-0.01707	0.580081	0.316347
2.2	1.3	2.75	0.346406	0.332922	-0.01776	0.616817	0.329014
2.2	1.3	3	0.355415	0.34123	-0.0182	0.64404	0.337123
2.2	1.3	3.25	0.361146	0.346507	-0.01848	0.664055	0.34227
2.2	1.3	3.5	0.364761	0.349831	-0.01866	0.678666	0.345512

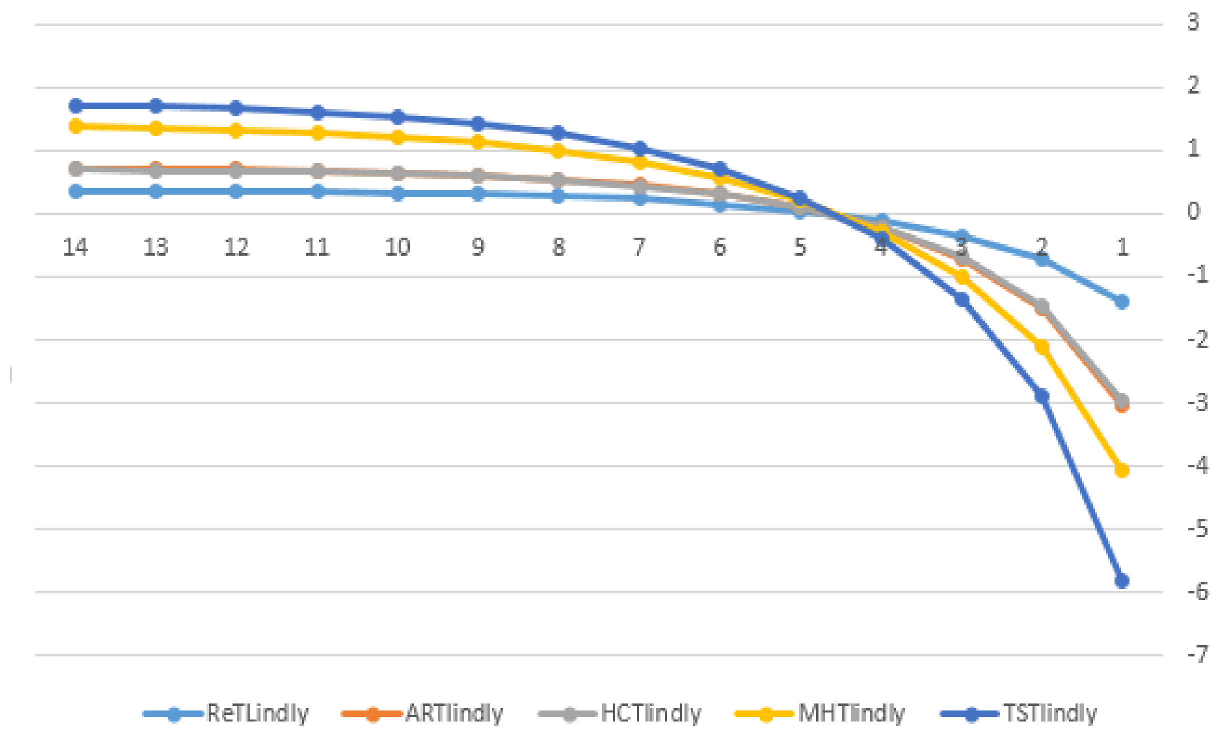


Figure 5. Entropies value for Table 4.

Table 5. Relative loss for $a = 0.5, c = 0.6$.

k	**Renyi**	**Armito**	**HC**	**MH**	**Ts**
0.25	2.513062	1.500018	1.500018	2.134917	1.860427
0.5	1.756721	1.33341	1.33341	1.470491	1.504138
0.75	1.314241	1.166794	1.166794	1.176008	1.230727
1	1.000233	1.000142	1.000137	1.000358	1.000184
1.25	0.756665	0.833482	0.833482	0.880491	0.797063
1.5	0.55774	0.666893	0.666893	0.79212	0.613507
1.75	0.389731	0.500504	0.500504	0.723642	0.444901
2	0.24447	0.334482	0.334482	0.668721	0.288287
2.25	0.116702	0.169024	0.169024	0.623561	0.14165
2.5	0.002851	0.004348	0.004348	0.585732	0.003552
2.75	-0.09963	-0.15931	-0.15931	0.553588	-0.12707
3	-0.19261	-0.3217	-0.3217	0.525973	-0.25101
3.25	-0.27753	-0.48259	-0.48259	0.50204	-0.36889
3.5	-0.35547	-0.6417	-0.6417	0.481156	-0.48122

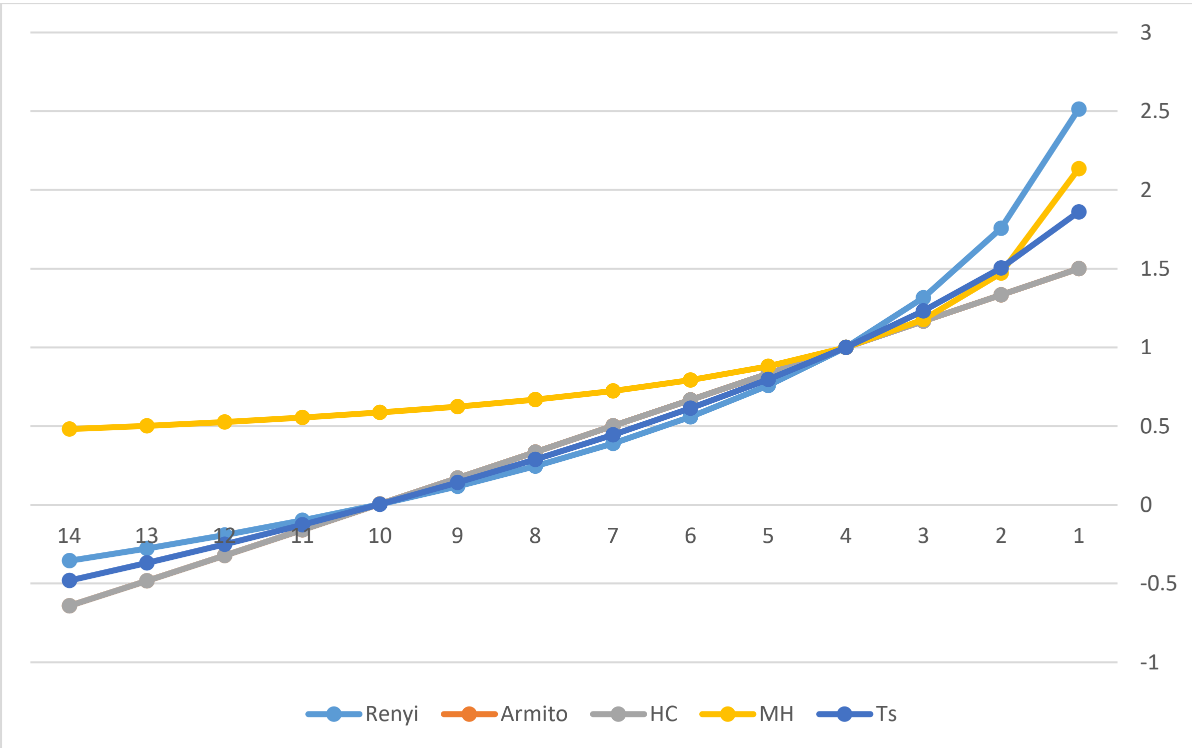


Figure 6. Entropies value for Table 5.

Table 6. Relative loss for $a = 2.2, c = 1.3$.

k	Renyi	Armito	HC	MH	Ts
0.25	1.81418	2.162004	2.162004	-1.5724	2.658681
0.5	1.420369	1.553362	1.553362	-0.44243	1.770016
0.75	1.204083	1.257202	1.257202	0.30624	1.353137
1	1.065646	1.080483	1.080483	0.85882	1.109583
1.25	0.972632	0.967057	0.967057	1.281552	0.955393
1.5	0.909337	0.892217	0.892217	1.608339	0.854595
1.75	0.866366	0.842463	0.842463	1.86075	0.788006
2	0.837456	0.809461	0.809461	2.054529	0.744024
2.25	0.81823	0.787721	0.787721	2.202048	0.715133
2.5	0.805598	0.773527	0.773527	2.313322	0.696307
2.75	0.797395	0.764348	0.764348	2.396494	0.684147
3	0.792126	0.758468	0.758467	2.458128	0.676362
3.25	0.788774	0.754733	0.754732	2.503442	0.671421
3.5	0.78666	0.752379	0.752379	2.536522	0.668308

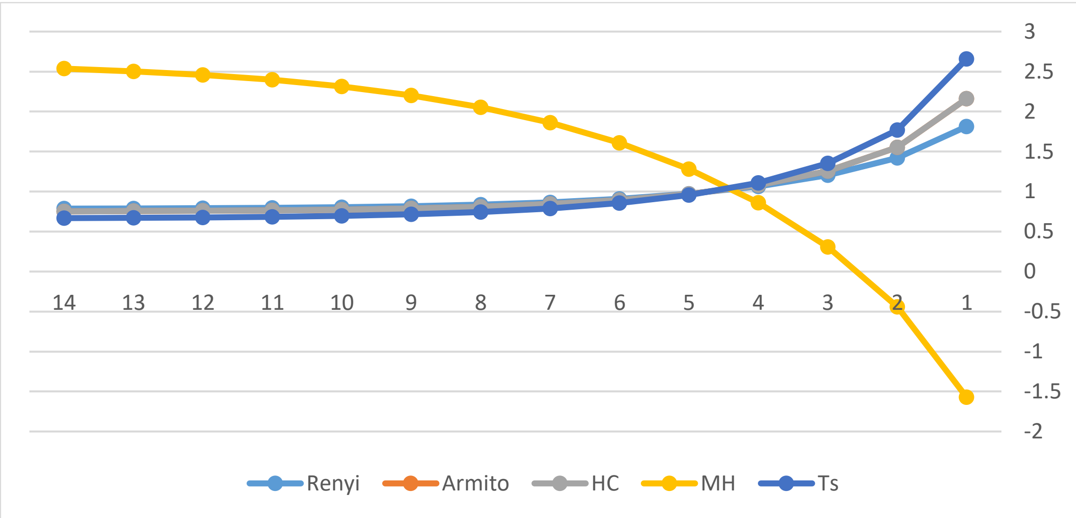


Figure 7. Entropies value for Table 6.

8. Conclusion

This study dealt with the use of the impulse loss function for five different measures of entropy functions for the Lindley and truncated Lindley distributions within a specific period. The results of the numerical analysis of these functions, supported by graphs, proved that their behavior was decreasing for the Lindley distribution with the increase of its parameter with a constant entropy index, while the truncated Lindley distribution was decreasing with a variation in its behavior with the variation of the decreasing values for these functions based on the increase in the value of the truncation parameter, while the relative loss function was in the same behavior for all values of the entropy functions except for the entropy function Mathai–Haubold, whose behavior was increasing at certain parameters relative to the loss function used in the analysis.

Authors' Contributions:

Khalaf H. Al-Habib, Moudher Kh. Abdal-hammed, Ahmed M. Salih, and Mundher A. Khaleel: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Visualization, Writing – original draft, Writing – review & editing. All authors contributed equally to this work and have read and approved the final manuscript.

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The data that supports the findings of this study are available within the article.

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The authors declare no conflict of interest.

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