Modern Journal of Statistics
2(2), 159–173
DOI: https://doi.org/10.64389/mjs.2026.02271
https://sphinxsp.org/journal/index.php/mjs

Research article

Designing of Multiple Dependent State Sampling Plan for Inverted Nadarajah-Haghighi distribution

Sd. Jilani[1] and Srinivasa Rao Gadde[2,*]

[1]Department of Statistics, QIS College of Engineering and Technology, Ongole, A.P. 523272, India; jilanisyed1992@gmail.com

[2]Department of Statistics, The University of Dodoma, Dodoma, P.O. Box 259, Tanzania; gaddesrao@gmail.com

***Correspondence:** gaddesrao@gmail.com

ARTICLE INFO

Keywords:
Inverted Nadarajah-Haghighi distribution
Consumer's risk
Producer's risk
Multiple dependent state sampling plan
Percentile ratio

Mathematics Subject Classification:
62G30, 60E05, 62B10

Important Dates:
Received: 31 January 2026
Revised: 30 May 2026
Accepted: 24 June 2026
Online: 30 June 2026

ABSTRACT

A novel approach to quality control is proposed through the development of a multiple dependent state (MDS) sampling plan for time-truncated life tests. The proposed plan assumes that product lifetimes follow an inverted Nadarajah–Haghighi (INH) distribution, with product quality evaluated using a specified percentile lifetime. The sampling plan parameters, namely the sample size, acceptance and rejection numbers and the required number of preceding lots for decision-making are determined using a two-point approach based on the operating characteristic (OC) curve. Extensive tables are constructed for different values of the shape parameter, and the performance of the plan is thoroughly analyzed. The practical applicability of the proposed sampling scheme is illustrated using real-world data and its efficiency is compared with existing sampling plans. The results demonstrate that the proposed plan provides improved protection for both producers and consumers.

1. Introduction

In recent years, the global market has witnessed a discernible decline in product quality. Rapid population growth and escalating consumer demand have compelled manufacturers to prioritize profit maximization, often at the cost of stringent quality control practices. This shift has led to severe consequences, including structural failures in buildings and the distribution of defective or unsafe medical products, both of which pose serious threats to human life.

To mitigate these challenges, it is imperative for manufacturers to implement effective quality control mechanisms, while consumers remain cautious in evaluating the products they purchase. Acceptance sampling emerges as a practical and efficient statistical approach to maintaining this balance. It enables manufacturers to decide whether to accept or reject production lots based on sample inspection, thereby reducing inspection costs without compromising quality standards. A key strength of acceptance sampling lies in its ability to balance two conflicting risks: the producer's risk (α), defined as the probability of rejecting a lot that meets quality standards and the consumer's risk (β), which represents the probability of accepting a lot of inferior quality. These risks are typically evaluated with respect to two benchmark quality levels, namely the Acceptable Quality Level (AQL) and the Limiting Quality Level (LQL).

The necessity of rigorous quality control spans several critical industries. In the construction sector, the use of substandard materials can undermine structural stability and endanger lives. In healthcare, faulty medical devices or ineffective pharmaceuticals may result in fatal consequences. Similarly, in the food industry, contaminated or inferior products can trigger widespread public health crises. By employing statistically sound acceptance sampling plans, manufacturers can significantly reduce these risks and ensure consistent product quality.

Acceptance sampling involves the random selection of a sample from a production lot, based on which a decision is made to accept or reject the entire lot. When designed using appropriate statistical models, such sampling plans minimize inspection effort while ensuring adequate protection for both producers and consumers. Moreover, acceptance sampling provides valuable feedback on the production process, enabling manufacturers to identify deficiencies and implement continuous quality improvements.

1.1 Multiple Dependent State Sampling Plan

In this section, the concept of MDS sampling plan was first introduced by Wortham and Baker (1976) [23]. Vaerst [22] modified the chain sampling plan proposed by Dodge [11] as MDS-1(c_1, c_2). Later, many authors have studied MDS sampling plan under various situations Govindaraju and subramani [13], Balamurali and Jun [6] and Balamurali et al. [7]. For more details on designing an attribute MDS sampling plans, please refer to Soundararajan and Vijayaraghavan [19], Subramani and Haridoss [20], Aslam et al. [5], Wu et al. [25], Wu et al. [24], Balamurali et al. [8], Balamurali et al. [9] and Aslam et al. [4]. Later, the concept of multiple dependent state sampling has been used in control chart design. For example, Aslam et al. ([2], [3]) developed control charts for exponential distributions using Multiple Dependent State (MDS) sampling plans. Jeyadurga and Balamurali [14] proposed an optimal multiple dependent state repetitive group (MDSRG) sampling plan for variables inspection. Rao et al. [17] developed MDS sampling plans under the exponentiated half logistic distribution, while Rao et al. [16] applied an MDS sampling approach to COVID-19 data using the exponentiated Weibull distribution. Aslam et al. [1] presented an economic design of a modified MDS sampling plan under various lifetime distributions. More recent developments include the work of Yen et al. [26], who introduced an MDS sampling plan based on one-sided process capability indices, and Geetha et al. [12], who proposed an MDS sampling inspection plan for variables. Furthermore, Jilani et al. [15] developed an MDS sampling plan for the Type-II generalized half logistic distribution, whereas Rao et al. [18] introduced an MDS sampling plan under the exponentiated Fréchet distribution.

Although considerable progress has been made in the development of MDS sampling plans, most existing studies focus on traditional lifetime distributions and quality measures based on mean life or conventional acceptance criteria. Very limited attention has been given to constructing MDS sampling plans using percentile lifetime ratios, particularly under flexible lifetime models capable of accommodating skewed lifetime data. In addition, to the best of our knowledge, no study has investigated an MDS sampling plan under the inverted Nadarajah–Haghighi (INH) distribution, despite its flexibility and suitability for modeling lifetime data with varying shapes.

Motivated by this research gap, the present study proposes a Multiple Dependent State (MDS) sampling plan based on percentile lifetime ratios under truncated life tests assuming INH distribution. The paper is organized as follows. Section 2 introduces the proposed MDS sampling plan under the INH distribution and Subsection 2.1 presents its operating procedure along with the derivation of the Operating Characteristic (OC) function. Subsection 2.2 explains the optimization procedure used to determine the optimal design parameters by minimizing the Average Sample Number (ASN) while satisfying producer's and consumer's risk requirements. Section 3 presents numerical results and discusses the generated tables for different values of the shape parameter. Section 4 compares the performance of the proposed MDS sampling plan with the conventional Single Sampling Plan (SSP) to evaluate its efficiency and practical applicability. Finally, Section 5 concludes the study and summarizes the key findings.

2. Invert ed Nadarajah-Haghighi distribution

In this study, the INH distribution is employed to model product lifetimes. The fundamental characteristics of the distribution, including its probability density function (PDF), cumulative distribution function (CDF) and quantile function (QF) are adopted from Tahir et al. [21]. Owing to its flexible shape and inherent skewness, the INH distribution is particularly suitable for modeling lifetime data in reliability and quality control applications. Furthermore, the availability of closed-form expressions for both the CDF and QF facilitates the development of multiple dependent state sampling plans based on percentile lifetime ratios.

Let t denote a random variable representing the product lifetime and assume that t follows an INH distribution, denoted by $t \sim INH(\gamma, \theta)$. Where, $\gamma > 0$ is the shape parameter, $\theta > 0$ is the scale parameter. The cumulative distribution function (CDF) of the INH distribution is given by:

$$F(x) = e^{1-\left(1+\theta x^{-1}\right)^{\gamma}}, \quad x \in R. \tag{2.1}$$

The corresponding probability density function (PDF) is expressed as:

$$f(x) = \gamma\theta x^{-2}\left(1+\theta x^{-1}\right)^{\gamma-1} e^{1-\left(1+\theta x^{-1}\right)^{\gamma}}. \tag{2.2}$$

The 100q-th percentile (quantile) of the INH distribution is obtained as:

$$t_q = \theta\eta_q, \qquad \text{where } \eta_q = \left((1-\log q)^{1/\gamma} - 1\right)^{-1}, \ u \in (0,1), \tag{2.3}$$

where 0 <q <1 represents the percentile level.

The median lifetime (50th percentile) of the INH distribution is obtained by substituting q=0.5 into the quantile function. Hence, the median lifetime is given by

$$M = \theta\left[(1+\log 2)^{1/\gamma} - 1\right]^{-1}.$$

If the lifetime of the product *t* as follows INH with terminating time t_0 and truncating time t_q^0, then $p = F(t_0)$. Experiment test termination t_0 time can be expressed as $t_0 = at_q^0$.

$$p = e^{1-\left(1+\left(t_q/t_q^0\left(\eta_q a\right)^{-1}\right)\right)^{\gamma}}. \tag{2.4}$$

2.1 Operating Procedure of the MDS Sampling Plan

The MDS sampling plan determines lot disposition based not only on the quality of the current lot but also on the acceptance status of the preceding m lots.

- n = sample size selected from the current lot,
- t_0 = predetermined experiment (termination) time,
- D = number of observed failures in the current sample before time t_0,
- c_1 = acceptance number,
- c_2 = rejection number, where $c_1 < c_2$,
- m = number of immediately preceding lots considered in the decision process.

The operating procedure of the proposed MDS plan is described below.

Step 1: Select a random sample of size n from the submitted lot and subject all units to a life test for a predetermined termination time t_0.

Step 2: Record the number of failed items occurring before the experiment termination time t_0. Let this observed number of failures be denoted by D.

Step 3: Make the lot disposition according to the following decision rules:

1. **Acceptance Rule:** Accept the current lot if $D \le c_1$.
2. **Rejection Rule:** Reject the current lot if $D > c_2$.
3. **Conditional Acceptance Rule:** If $c_1 < D \le c_2$.

Then the decision depends on the quality history of the preceding m lots. The current lot is accepted only if each of the previous m submitted lots recorded failures not exceeding the acceptance number c_1, that is,

$$D_j \le c_1, \quad j=1, 2, \dots, m$$

where D_j denotes the observed number of failures in the j[th] previous lot before termination time t_0. Otherwise, the current lot is rejected.

Thus, our proposed multiple dependent state sampling plan is defined by four key parameters are c_1, c_2, m and n. where c_1 is the maximum number of allowable failure items for unconditional acceptance, c_2 is the maximum number of additional failure items for conditional acceptance, the m number of previous lots required to make a decision, and n sample number.

The SSP can be viewed as a special case of the MDS sampling plan. MDS sampling plan generalizes SSP and this relationship is evident as MDS sampling plan reduces to SSP when either the number of previous lots (m) approaches infinity or when the acceptance numbers c_1 and c_2 converge to a single value c.

The Operating Characteristic (OC) function represents the probability of accepting a submitted lot under the proposed MDS sampling plan. It measures the discriminatory power of the sampling procedure by relating the probability of lot acceptance to the underlying product quality level.

According to the operating rules of the proposed MDS plan, a lot is accepted under two possible situations:

1. The current lot is accepted directly if the observed number of failures satisfies $D \le c_1$;
2. If the observed failures fall within the conditional region $c_1 < D \le c_2$, the current lot is accepted only when each of the previous m lots had failure counts not exceeding c_1.

Therefore, the probability of accepting a lot is expressed as:

$$P_a(p) = p(D \le c_1) + p(c_1 < D \le c_2)\left(p(D \le c_1)\right)^m \quad . \tag{2.5}$$

Equation (2.5) consists of two components: the first term represents direct acceptance of the current lot, whereas the second term represents conditional acceptance based on the acceptance performance of the previous m lots.

Substituting the binomial probabilities into Equation (2.5), the OC function becomes:

$$P_a(p) = \sum_{D=0}^{c_1} \binom{n}{D} p^D \ (1-p)^{n-D} + \sum_{D=c_1+1}^{c_2} \binom{n}{D} p^D \ (1-p)^{n-D} \left[\sum_{D=0}^{c_1} \binom{n}{D} p^D \ (1-p)^{n-D} \right]^m . \tag{2.6}$$

Equation (2.6) gives the explicit form of the OC function under the binomial assumption and is used to compute the probability of accepting lots for different quality levels.

2.2 Designing methodology

Sampling plans are generally designed to minimize the Average Sample Number (ASN) because a lower ASN reduces inspection time, testing effort and associated costs while maintaining the required quality protection levels. Therefore, a sampling plan with a smaller ASN is considered more efficient and economically desirable.

For the proposed MDS sampling plan, only one sample of size n is drawn from each submitted lot. Consequently, the ASN is equivalent to the sample size n. Thus, determining the optimal plan reduces to finding the minimum sample size that satisfies the specified producer's and consumer's risk requirements

Accordingly, the optimal design parameters (n, c_1, c_2, m) are obtained by solving the following optimization problem.

$$\begin{aligned} &\text{Minimum } ASN(p) = n \\ &\text{subject to } p_a(p_1) \ge 1-\alpha \\ &\qquad p_a(p_2) \le \beta \\ &\qquad n > 1, m \ge 1, \\ &\qquad c_2 > c_1 \ge 0 \end{aligned} \tag{2.7}$$

$$P_a(p_1) = \sum_{D=0}^{c_1} \binom{n}{D} p_1^D \ (1-p_1)^{n-D} + \sum_{D=c_1+1}^{c_2} \binom{n}{D} p_1^D \ (1-p_1)^{n-D} \left[\sum_{D=0}^{c_1} \binom{n}{D} p_1^D \ (1-p_1)^{n-D} \right]^m . \tag{2.8}$$

$$P_a(p_2) = \sum_{D=0}^{c_1} \binom{n}{D} p_2^D \ (1-p_2)^{n-D} + \sum_{D=c_1+1}^{c_2} \binom{n}{D} p_2^D \ (1-p_2)^{n-D} \left[\sum_{D=0}^{c_1} \binom{n}{D} p_2^D \ (1-p_2)^{n-D} \right]^m . \tag{2.9}$$

where:

- Pa(p) denotes the OC function of the proposed MDS sampling plan;
- p_1 represents the probability of failure corresponding to the acceptable quality level (AQL);
- p_2 represents the probability of failure corresponding to the limiting quality level (LQL);
- α is the producer's risk, defined as the probability of rejecting a good-quality lot;
- β is the consumer's risk, defined as the probability of accepting a poor-quality lot;

Equation (2.7) minimizes the required inspection effort by selecting the smallest feasible sample size. Equation (2.8) ensures that good quality lots are accepted with probability at least $1-\alpha$, while Equation (2.9) restricts the acceptance probability of poor-quality lots to at most β. The feasible combinations of (n, c_1, c_2, m) satisfying these constraints are evaluated and the combination yielding the minimum sample size is selected as the optimal sampling plan.

This method assesses quality level by calculating the ratio of true percentile lifetime to true lifetime t_q / t_q^0. By applying the percentile ratio concept, producers can enhance product quality. Notably, at the consumer's risk, the percentile ratio $\left(t_q / t_q^0\right)$ equals 1. However, at the producer's risk, this ratio varies, taking values such as $t_q / t_q^0 = 2$, 2.5, 3.0, 3.5 and 4.

Table 1. Best parameters of the proposed MDS plan for INH distribution with γ=0.5.

β	t_q/t_q^0	a=0.5							a=1.0						
		n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN	n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN
0.25	2.0	20	4	14	2	0.9508	0.2483	20	22	8	13	1	0.9539	0.2453	22
	2.5	12	2	11	3	0.9521	0.2424	12	12	4	11	2	0.9525	0.2241	12
	3.0	8	1	7	3	0.9510	0.2490	8	10	3	9	3	0.9518	0.1761	10
	3.5	8	1	7	6	0.9624	0.2388	8	7	2	6	5	0.9535	0.2270	7
	4.0	8	1	7	6	0.9827	0.2388	8	5	1	4	2	0.9521	0.2150	5
0.10	2.0	36	7	14	2	0.9562	0.0997	36	35	13	23	2	0.9521	0.0946	35
	2.5	22	3	8	1	0.9540	0.0999	22	20	6	11	1	0.9514	0.0975	20
	3.0	16	2	12	2	0.9630	0.0943	16	14	4	13	2	0.9593	0.0971	14
	3.5	13	1	5	1	0.9599	0.0988	13	12	3	11	2	0.9621	0.0779	12
	4.0	12	1	11	4	0.9526	0.0759	12	9	2	8	3	0.9512	0.0905	9
0.05	2.0	48	9	19	2	0.9505	0.0457	48	45	16	21	1	0.9539	0.0488	45
	2.5	29	4	9	1	0.9576	0.0480	29	26	8	11	1	0.9525	0.0469	26
	3.0	23	3	13	4	0.9540	0.0451	23	18	5	10	2	0.9561	0.0498	18
	3.5	19	2	12	4	0.9537	0.0392	19	14	3	7	1	0.9516	0.0452	14
	4.0	14	1	11	2	0.9562	0.0430	14	11	2	5	1	0.9516	0.0480	11
0.01	2.0	73	13	18	1	0.9585	0.0099	73	69	24	31	1	0.9560	0.0095	69
	2.5	44	6	12	1	0.9596	0.0099	44	40	12	16	1	0.9558	0.0093	40
	3.0	34	4	14	2	0.9606	0.0090	34	28	7	14	1	0.9544	0.0098	28
	3.5	25	2	6	1	0.9543	0.0092	25	22	5	8	1	0.9631	0.0096	22
	4.0	24	2	12	4	0.9541	0.0095	24	19	4	14	2	0.9642	0.0097	19

Tables 1-5 present the optimal parameters for a MDS sampling plan tailored to the INH distribution, assuming known shape parameters γ= 0.5, 1.0, 1.5, 2.0 and 2.5 under truncated life tests. The parameters were determined considering a producer's

Table 2. Best parameters of the proposed MDS plan for INH distribution with γ=1.0.

β	t_q/t_q^0	*a*=0.5							*a*=1.0						
		n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN	n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN
0.25	2.0	11	1	10	2	0.9596	0.2283	11	12	4	11	2	0.9542	0.2241	12
	2.5	6	0	1	1	0.9588	0.2413	6	7	2	6	5	0.9510	0.2270	7
	3.0	5	0	4	6	0.9715	0.2374	5	5	1	4	4	0.9514	0.1885	5
	3.5	5	0	4	6	0.9919	0.2374	5	5	1	4	6	0.9783	0.1875	5
	4.0	5	0	4	6	0.9979	0.2374	5	3	0	2	1	0.9688	0.2188	3
0.10	2.0	20	2	12	3	0.9582	0.0920	20	20	6	11	1	0.9534	0.0975	20
	2.5	15	1	11	6	0.9697	0.0802	15	12	3	11	2	0.9596	0.0779	12
	3.0	9	0	8	3	0.9542	0.0755	9	9	2	8	6	0.9596	0.0898	9
	3.5	9	0	8	6	0.9765	0.0751	9	7	1	6	4	0.9507	0.0625	7
	4.0	9	0	8	6	0.9934	0.0751	9	7	1	6	6	0.9775	0.0625	7
0.05	2.0	24	2	4	1	0.9529	0.0480	24	26	8	11	1	0.9548	0.0469	26
	2.5	18	1	11	5	0.9534	0.0395	18	16	4	14	3	0.9507	0.0385	16
	3.0	11	0	10	2	0.9534	0.0439	11	11	2	10	2	0.9585	0.0338	11
	3.5	11	0	10	6	0.9666	0.0422	11	8	1	7	2	0.9567	0.0363	8
	4.0	11	0	10	6	0.9904	0.0422	11	8	1	7	6	0.9649	0.0352	8
0.01	2.0	38	3	8	1	0.9548	0.0098	38	40	12	16	1	0.9584	0.0093	40
	2.5	24	1	3	1	0.9654	0.0100	24	22	5	8	1	0.9601	0.0096	22
	3.0	24	1	11	6	0.9849	0.0090	24	17	3	13	2	0.9561	0.0064	17
	3.5	17	0	10	3	0.9588	0.0075	17	14	2	12	4	0.9516	0.0065	14
	4.0	17	0	10	6	0.9788	0.0075	17	11	1	10	2	0.9596	0.0059	11

Table 3. Best parameters of the proposed MDS plan for INH distribution with γ=1.5.

β	t_q/t_q^0	*a*=0.5							*a*=1.0						
		n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN	n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN
0.25	2.0	12	1	11	6	0.9816	0.2140	12	10	3	5	1	0.9631	0.2494	10
	2.5	6	0	5	6	0.9772	0.2192	6	5	1	4	2	0.9563	0.2150	5
	3.0	6	0	5	6	0.9973	0.2192	6	5	1	4	6	0.9764	0.1875	5
	3.5	6	0	5	6	1.000	0.2192	6	3	0	2	2	0.9538	0.1367	3
	4.0	6	0	5	6	1.0000	0.2192	6	3	0	2	6	0.9552	0.1250	3
0.10	2.0	16	1	11	6	0.9537	0.0979	16	17	5	15	2	0.9529	0.0765	17
	2.5	10	0	9	5	0.9521	0.0797	10	9	2	8	3	0.9568	0.0905	9
	3.0	10	0	9	6	0.9929	0.0797	10	7	1	6	3	0.9576	0.0627	7
	3.5	10	0	9	6	0.9993	0.0797	10	7	1	6	6	0.9846	0.0625	7
	4.0	10	0	9	6	0.9999	0.0797	10	4	0	3	3	0.9570	0.0627	4
0.05	2.0	20	1	11	2	0.9614	0.0446	20	22	6	13	1	0.9583	0.0480	22
	2.5	12	0	10	3	0.9559	0.0481	12	11	2	5	1	0.9571	0.0480	11
	3.0	12	0	10	6	0.9901	0.0480	12	8	1	7	2	0.9532	0.0363	8

	3.5	12	0	10	6	0.9990	0.0480	12	8	1	7	6	0.9756	0.0352	8
	4.0	12	0	10	6	0.9999	0.0480	12	5	0	4	2	0.9545	0.0322	5
0.01	2.0	35	2	12	4	0.9574	0.0086	35	33	9	15	1	0.9651	0.0092	33
	2.5	19	0	2	1	0.9601	0.0095	19	17	3	8	1	0.9509	0.0095	17
	3.0	19	0	10	6	0.9772	0.0082	19	14	2	12	3	0.9573	0.0065	14
	3.5	19	0	10	6	0.9977	0.0082	19	11	1	10	4	0.9502	0.0059	11
	4.0	19	0	10	6	0.9998	0.0082	19	7	0	2	1	0.9538	0.0095	7

Table 4. Best parameters of the proposed MDS plan for INH distribution with γ=2.0.

β	t_q/t_q^0	*a*=0.5							*a*=1.0						
		n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN	n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN
0.25	2.0	6	0	5	4	0.9521	0.2486	6	10	3	9	3	0.9515	0.1761	10
	2.5	6	0	5	6	0.9949	0.2460	6	5	1	4	4	0.9504	0.1885	5
	3.0	6	0	5	6	0.9998	0.2460	6	3	0	2	1	0.9588	0.2188	3
	3.5	6	0	5	6	1.0000	0.2460	6	3	0	2	4	0.9554	0.1252	3
	4.0	6	0	5	6	1.0000	0.2460	6	3	0	2	6	0.9803	0.1250	3
0.10	2.0	18	1	11	6	0.9846	0.0853	18	14	4	13	2	0.9590	0.0971	14
	2.5	10	0	9	6	0.9869	0.0965	10	9	2	8	6	0.9585	0.0898	9
	3.0	10	0	9	6	0.9994	0.0965	10	7	1	6	6	0.9638	0.0625	7
	3.5	10	0	9	6	1.0000	0.0965	10	4	0	3	2	0.9581	0.0659	4
	4.0	10	0	9	6	1.0000	0.0965	10	4	0	3	6	0.9674	0.0625	4
0.05	2.0	21	1	11	6	0.9744	0.0481	21	18	5	10	2	0.9557	0.0498	18
	2.5	13	0	10	6	0.9789	0.0478	13	11	2	10	2	0.9574	0.0338	11
	3.0	13	0	10	6	0.9990	0.0478	13	8	1	7	5	0.9515	0.0352	8
	3.5	13	0	10	6	1.0000	0.0478	13	5	0	2	1	0.9655	0.0459	5
	4.0	13	0	10	6	1.0000	0.0478	13	5	0	4	6	0.9524	0.0313	5
0.01	2.0	29	1	11	3	0.9594	0.0098	29	28	7	14	1	0.9540	0.0098	28
	2.5	20	0	10	6	0.9556	0.0093	20	17	3	13	2	0.9546	0.0064	17
	3.0	20	0	10	6	0.9976	0.0093	20	11	1	5	1	0.9648	0.0088	11
	3.5	20	0	10	6	0.9999	0.0093	20	11	1	10	6	0.9759	0.0059	11
	4.0	20	0	10	6	1.0000	0.0093	20	7	0	6	3	0.9502	0.0078	7

Table 5. Best parameters of the proposed MDS plan for INH distribution with γ=2.5.

β	t_q/t_q^0	*a*=0.5							*a*=1.0						
		n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN	n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN
0.25	2.0	7	0	6	6	0.9550	0.2119	7	8	2	5	1	0.9588	0.2473	8
	2.5	7	0	6	6	0.9981	0.2119	7	5	1	4	6	0.9504	0.1875	5
	3.0	7	0	6	6	1.0000	0.2119	7	3	0	2	1	0.9696	0.2188	3
	3.5	7	0	6	6	1.0000	0.2119	7	3	0	2	6	0.9609	0.1250	3
	4.0	7	0	6	6	1.0000	0.2119	7	3	0	2	6	0.9900	0.1250	3

0.10	2.0	11	0	10	2	0.9588	0.0942	11	14	4	13	3	0.9573	0.0904	14
	2.5	11	0	10	6	0.9956	0.0873	11	7	1	3	1	0.9616	0.0898	7
	3.0	11	0	10	6	0.9999	0.0873	11	7	1	6	6	0.9786	0.0625	7
	3.5	11	0	10	6	1.0000	0.0873	11	4	0	3	4	0.9531	0.0625	4
	4.0	11	0	10	6	1.0000	0.0873	11	4	0	3	6	0.9830	0.0625	4
0.05	2.0	15	0	2	1	0.9599	0.0491	15	18	5	15	3	0.9555	0.0482	18
	2.5	14	0	10	6	0.9930	0.0449	14	11	2	10	4	0.9502	0.0327	11
	3.0	14	0	10	6	0.9999	0.0449	14	8	1	7	6	0.9664	0.0352	8
	3.5	14	0	10	6	1.0000	0.0449	14	5	0	4	2	0.9606	0.0322	5
	4.0	14	0	10	6	1.0000	0.0449	14	5	0	4	6	0.9747	0.0313	5
0.01	2.0	31	1	11	6	0.9720	0.0090	31	25	6	11	1	0.9524	0.0098	25
	2.5	21	0	10	6	0.9852	0.0095	21	14	2	6	1	0.9556	0.0090	14
	3.0	21	0	10	6	0.9997	0.0095	21	11	1	10	2	0.9613	0.0059	11
	3.5	21	0	10	6	1.0000	0.0095	21	7	0	2	1	0.9600	0.0095	7
	4.0	21	0	10	6	1.0000	0.0095	21	7		6	6	0.9550	0.0078	7

Table 6. Best parameters of the proposed MDS plan for INH distribution with γ=0.4240.

β	t_q/t_q^0	*a*=0.5							*a*=1.0						
		n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN	n	c_1	c_2	m	$P_a(p_1)$	$P_a(p_2)$	ASN
0.25	2.0	26	6	16	2	0.9567	0.2476	26	25	10	20	2	0.9566	0.2476	25
	2.5	15	3	13	3	0.9519	0.2407	15	14	5	13	2	0.9565	0.2474	14
	3.0	12	2	11	3	0.9533	0.2054	12	10	3	5	1	0.9558	0.2494	10
	3.5	8	1	7	2	0.9566	0.2453	8	8	2	5	1	0.9581	0.2473	8
	4.0	8	1	7	4	0.9578	0.2119	8	7	2	6	5	0.9541	0.2270	7
0.10	2.0	45	10	17	2	0.9594	0.0999	45	43	16	2	1	0.9569	0.0983	43
	2.5	27	5	15	2	0.9581	0.0931	27	25	8	15	1	0.9529	0.0987	25
	3.0	19	3	13	2	0.9616	0.0989	19	18	5	15	1	0.9512	0.0939	18
	3.5	15	2	12	2	0.9628	0.0980	15	14	4	13	3	0.9562	0.0904	14
	4.0	12	1	4	1	0.9517	0.0978	12	12	3	11	2	0.9628	0.0779	12
0.05	2.0	58	12	17	1	0.9538	0.0462	58	54	20	25	1	0.9540	0.0495	54
	2.5	35	6	10	1	0.9600	0.0473	35	31	10	1	1	0.9533	0.0469	31
	3.0	26	4	14	2	0.9611	0.0457	26	23	7	17	2	0.9629	0.0486	23
	3.5	22	3	13	3	0.9590	0.0424	22	18	5	15	3	0.9543	0.0482	18
	4.0	18	2	12	2	0.9631	0.0402	18	14	3	7	1	0.9524	0.0452	14
0.01	2.0	89	18	26	1	0.9592	0.0099	89	54	20	25	1	0.9540	0.0495	54
	2.5	53	9	12	1	0.9506	0.0099	53	31	10	14	1	0.9533	0.0469	31
	3.0	37	5	9	1	0.9527	0.0093	37	23	7	17	2	0.9629	0.0486	23
	3.5	32	4	14	2	0.9592	0.0093	32	18	5	15	3	0.9543	0.0482	18
	4.0	28	3	13	2	0.9608	0.0080	28	14	3	7	1	0.9524	0.0452	14

risk $\alpha = 0.05$ and consumer's risks $\beta = 0.25, 0.10, 0.05$ and 0.01, with termination ratios (a) ranging from 0.5 to 1.0 at the 50th percentile value. Additionally, Table 6 provides the optimal parameters for the MDS sampling plan when the shape parameters are estimated $\hat{\gamma} = 0.4240$ based on real-life time data, using termination ratios of a=0.5 and a=1.0.

The results in Tables 1-6 highlight key findings from the proposed MDS sampling plan.

(1) When other parametric combinations are fixed values of β, t_q / t_q^0 and γ when the value of a is increased from 0.5 to 1.0. In some parameter settings, the sample size increases, whereas in others it decreases. This indicates that the relationship between the acceptance parameter and sample size depends on the interaction among the plan parameters.
(2) When producer and consumer risks are fixed, the ASN value falls as the percentile ratio increases. It suggests that in order to obtain more accurate results with a higher percentile ratio, a smaller ASN is required.
(3) It is interesting to note that β value decreases from 0.25 to 0.01 the sample size n is decreases when all other parametric combinations are same.
(4) Additionally, when all other parameter combinations are fixed, the sample size decreases as the value of the corresponding design parameter increases.
(5) The findings showed that, when other parametric combinations are fixed, an increase in the quantile ratio also increase producer's probability of lot acceptance.

3. Application of the Proposed MDS Sampling Plan with Real Data Set

To demonstrate the practical applicability of the proposed MDS sampling plan, a real-life lifetime dataset consisting of 50 observations is considered. The dataset was previously reported by Zang et.al [27] and has been widely used in reliability and lifetime modeling studies. The observations represent positive continuous lifetime measurements and exhibit substantial variability with several relatively large values, indicating a positively skewed distribution. The observed data are: 1.120, 0.170, 0.640, 4.320, 1.220, 0.370, 1.160, 1.420, 0.090, 1.670,0.130, 0.250, 0.080, 0.040, 2.350, 0.200, 0.780, 0.340, 1.020, 0.170, 1.760, 2.390, 0.500, 1.350,3.360, 0.450, 0.900, 2.920, 6.530, 1.620, 7.460, 3.190, 2.490, 1.400, 7.490, 0.570, 0.140, 0.630,5.230, 0.710, 0.680, 0.120, 0.090,3.470,5.930,1.820,4.200,7.290,3.130,3.410.

The INH distribution is selected for this application because of its flexibility in modeling positively skewed lifetime data and its ability to accommodate different hazard rate behaviors. In addition, the INH distribution provides closed-form expressions for the CDF and QF, which are required for constructing percentile-based acceptance sampling plans.

Since the proposed MDS plan is developed using percentile lifetime ratios as the quality parameter, selecting a distribution that accurately captures tail behavior and quantile characteristics is essential. The INH distribution is therefore appropriate because its quantile structure directly supports the estimation of percentile-based quality measures used in the sampling design.

The maximum likelihood estimate of INH distribution for the above data are $\hat{\theta} = 2.3177$ and $\hat{\gamma} = 0.4240$. The Kolmogorov-Smirnov test is used to assess the goodness of fit, and it is found that the data set Kolmogorov-Smirnov statistic is D=0.11407 with a p-value of 0.5334 and Loglikelihood = -87.12821 AIC = 178.2564 BIC =182.0805. To demonstrate the INH goodness of fit, Figure 1 displays the empirical and theoretical PDFs, empirical and theoretical CDFs, Q-Q plots and P-P plots.

Suppose that lifetime of the products follows INH distribution with shape parameter $\hat{\gamma} = 0.4240$. Assuming that, $\alpha = 0.05$, $\beta = 0.25$, $a = 0.5$ and t_q / t_q^0 =2, so from Table 7 the design parameters are $n = 26$, $c_1 = 6, c_2 = 16$, and $m = 2$. Thus, the plan can be implemented as follows: take a random sample of size 30 from the current lot. Accept the lot if 6 failures occur before 1.16. If more than 16 failures the lot will be rejected. If the number of failures as 6 to 16, the disposition of the lot will be dependent until the next preceding 6 lots are tested. If the preceding 6 lots are accepted, then accept the lot. Otherwise, reject lot and next lot.

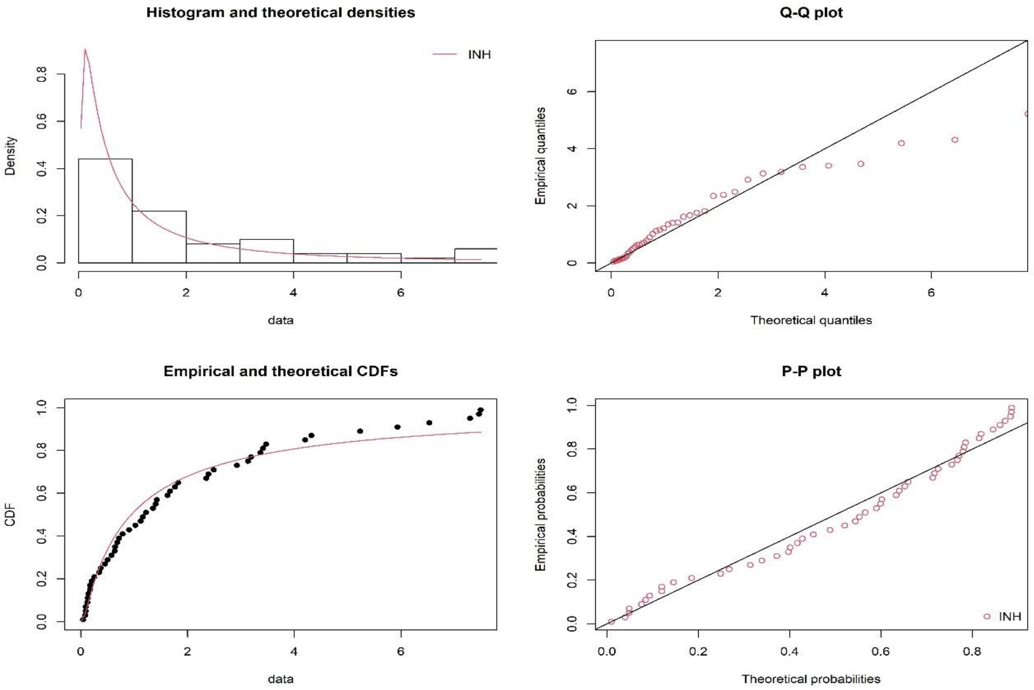

Figure 1: The empirical and theoretical PDFs, empirical and theoretical CDFs, Q-Q plots and P-P plots of the real-life time data as shown in graphs.

Figure 1 illustrates the graphical evaluation of the goodness-of-fit of the INH distribution to the real lifetime dataset using four diagnostic tools: the histogram with fitted theoretical density, empirical and theoretical CDFs, Q–Q plot and P–P plot. The histogram together with the fitted density curve indicates that the lifetime data are positively skewed with a long right tail and the INH distribution closely follows the observed data pattern, demonstrating its flexibility in modeling lifetime behaviour. The Q–Q plot shows that the empirical quantiles are generally close to the theoretical quantiles, especially in the lower and middle regions, indicating a satisfactory fit of the INH model to the data. In addition, the empirical and theoretical CDFs exhibit strong agreement across most of the observation range, further supporting the adequacy of the fitted model. The P–P plot also reveals that the empirical probabilities closely align with the theoretical probabilities along the reference line, providing further evidence that the INH distribution is appropriate for representing the real lifetime dataset.

4. Comparative Study

When quality control is performed using INH distribution, a comparison is made between a SSP and an MDS sampling plan. The OC curve illustrates the efficiency of the plan. The curve shows the difference in probabilities of accepting a good lot versus rejecting a bad lot. Table 7 shows the proposed MDS sampling plan outperforms the SSP when the underlying data distribution is assumed to be an INH distribution with a quantile ratio t_q / t_q^0 = 2, 2.5, 3.0, 3.5 and 4 for each consumer's risk $\beta = 0.25, 0.10, 0.05, 0.01$ while keeping producer's risk at $\alpha = 0.05$. The comparison mainly focuses on the sample size n and the probability of acceptance P(p1). For several set parameters, the proposed MDS sampling plan has a smaller acceptance sample size than the existing single-sampling plan (see Table 7). The plan parameters for the MDS sampling plan are $n = 26, c_1 = 6, c_2 = 16,$ and $m = 2$, whereas the plan parameters for the SSP are n=46 c=12, with corresponding acceptance probabilities of 0.9567 and 0.9535, respectively, when $\beta = 0.25$, α=0.5, γ=0.420, $a = 0.5$ and $t_q / t_q^0 = 2$. The MDS sampling plan has a smaller acceptance sample size than SSP. As the quantile ratio t_q / t_q^0 increased, the acceptance sample size decreased for both sampling methods. Similarly, the plan parameters for the MDS sampling plan are $n = 25, c_1 = 10, c_2 = 20,$ and $m = 2$, while the plan parameters for the SSP are n=44, c=19, with corresponding probability of acceptance of 0.9566 and 0.9514, respectively, when $\beta = 0.25$, α=0.5, γ=0.420, $a = 1.0$ and $t_q / t_q^0 = 2$. The MDS sampling plan has a smaller acceptance sample size than SSP. Figure 2 also shows OC curves for comparing the MDS sampling plan to the SSP under INH. Figure 2 are $n = 26, c_1 = 6, c_2 = 16,$ and $m = 2$ the parameters of the proposed MDS sampling plan and SSP with n=46 c=12. In terms of sample size, the MDS sampling plan under INH distribution is significantly more efficient than SSP.

Table 7: Comparison of optimal parameters of the proposed MDS sampling plan and SSP for INH distribution with $\hat{\gamma}$=0.4240.

β	t_q/t_q^0	a=0.5								a=1.0							
		MDS					SSP			MDS					SSP		
		n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$
0.25	2.0	26	6	16	2	0.9567	46	12	0.9535	25	10	20	2	0.9566	44	19	0.9514
	2.5	15	3	13	3	0.9519	29	7	0.9648	14	5	13	2	0.9565	27	11	0.9599
	3.0	12	2	11	3	0.9533	19	4	0.9522	10	3	5	1	0.9558	18	7	0.9625
	3.5	8	1	7	2	0.9566	15	3	0.9622	8	2	5	1	0.9581	14	5	0.9565
	4.0	8	1	7	4	0.9578	15	3	0.9796	7	2	6	5	0.9541	12	4	0.9569
0.10	2.0	45	10	17	2	0.9594	73	18	0.9580	43	16	2	1	0.9569	68	28	0.9505
	2.5	27	5	15	2	0.9581	45	10	0.9648	25	8	15	1	0.9529	39	15	0.9575
	3.0	19	3	13	2	0.9616	31	6	0.9550	18	5	15	1	0.9512	28	10	0.9588
	3.5	15	2	12	2	0.9628	23	4	0.9526	14	4	13	3	0.9562	21	7	0.9573
	4.0	12	1	4	1	0.9517	19	3	0.9541	12	3	11	2	0.9628	19	6	0.9651
0.05	2.0	58	12	17	1	0.9538	92	22	0.9581	54	20	25	1	0.9540	87	35	0.9505
	2.5	35	6	10	1	0.9600	53	11	0.9510	31	10	1	1	0.9533	51	19	0.9589
	3.0	26	4	14	2	0.9611	38	7	0.9517	23	7	17	2	0.9629	35	12	0.9562
	3.5	22	3	13	3	0.9590	30	5	0.9552	18	5	15	3	0.9543	28	9	0.9613
	4.0	18	2	12	2	0.9631	26	4	0.9620	14	3	7	1	0.9524	23	7	0.9650
0.01	2.0	89	18	26	1	0.9592	132	30	0.9539	54	20	25	1	0.9540	125	49	0.9550
	2.5	53	9	12	1	0.9506	80	16	0.9614	31	10	14	1	0.9533	73	26	0.9575
	3.0	37	5	9	1	0.9527	57	10	0.9591	23	7	17	2	0.9629	52	17	0.9597
	3.5	32	4	14	2	0.9592	45	7	0.9580	18	5	15	3	0.9543	40	12	0.9573
	4.0	28	3	13	2	0.9608	41	6	0.9719	14	3	7	1	0.9524	33	9	0.9507

From Table 8, it is evident that the proposed MDS sampling plan consistently exhibits lower ASN values than the conventional SSP across most parameter combinations considered. For example, when $\alpha = 0.05$, $\beta = 0.10$, $\gamma = 0.4240$, $a = 0.5$ and $t_q / t_q^0 = 2$, the proposed MDS sampling plan requires an ASN of 45, whereas the SSP requires an ASN of 73. Similarly, for another parameter combination with $\alpha = 0.05$, $\beta = 0.10$, $\gamma = 0.4240$, $a = 1.0$ and $t_q / t_q^0 = 2$ under the second case, the proposed MDS plan yields an ASN of 43 compared to an ASN of 68 for the SSP. These results demonstrate that the proposed

MDS sampling plan under the INH lifetime distribution achieves the required quality protection with a smaller sample size, making it more efficient and economical than the conventional SSP in terms of inspection effort and sampling cost.

Table 8. ASN values MDS and SSP for the INH Distribution with $\hat{\gamma}$=0.4240, when $a = 0.5$ and $a = 1.0$.

β	t_q/t_q^0	$a = 0.5$		$a = 1.0$	
		MDS	SSP	MDS	SSP
0.25	2.0	26	46	25	44
	2.5	15	29	14	27
	3.0	12	19	10	18
	3.5	8	15	8	14
	4.0	8	15	7	12
0.10	2.0	45	73	43	68
	2.5	27	45	25	39
	3.0	19	31	18	28
	3.5	15	23	14	21
	4.0	12	19	12	19
0.05	2.0	58	92	54	87
	2.5	35	53	31	51
	3.0	26	38	23	35
	3.5	22	30	18	28
	4.0	18	26	14	23
0.01	2.0	89	132	54	125
	2.5	53	80	31	73
	3.0	37	57	23	52
	3.5	32	45	18	40
	4.0	41	41	14	33

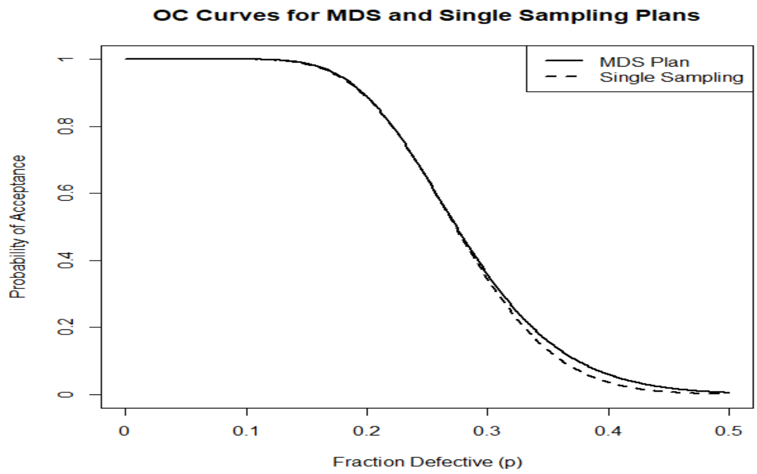

Figure 2: OC curve for MDS sampling plan and single sampling plan for INH distribution

Figure 2 shows the operating characteristic (OC) curves of the proposed MDS sampling plan and the conventional single sampling plan under the INH distribution. The OC curve represents the probability of accepting a lot at different levels of fraction defective (p). The results indicate that the probability of lot acceptance decreases as the fraction defective increases for both plans. At lower defect levels, both plans show similar acceptance performance; however, the proposed MDS plan provides a more gradual and effective transition from acceptance to rejection as defect levels increase.

5. Conclusions

In this study, a MDS sampling plan was developed under the INH distribution. The proposed method evaluates product quality using the ratio of the actual percentile lifetime to the specified percentile lifetime under truncated life tests. The optimal design parameters were obtained using the two-point approach based on the OC curve while satisfying both producer's and consumer's risk requirements. The numerical results show that the proposed MDS sampling plan performs efficiently in terms of ASN, resulting in reduced inspection effort. A comparative analysis with the conventional SSP was also carried out using OC curves. The findings indicate that the proposed MDS sampling plan provides better inspection performance by maintaining adequate protection for both producers and consumers while requiring fewer sample units. Therefore, the proposed approach offers a practical and efficient alternative for quality control and reliability applications involving lifetime data.

Authors' Contributions:

Conceptualization, G. S.R and S.J.; Methodology, G. S.R.; Software, G. S.R.; Validation, G. S.R. and S.J.; Formal Analysis, G. S.R.; Investigation, G. S.R.; Resources, G. S.R.; Data Curation, S.J.; Writing – Original Draft Preparation, S.J.; Writing – Review & Editing, G. S.R.; Visualization, G. S.R.; Supervision, G. S.R..

Data Availability Statement:

The data that supports the findings of this study are available within the article.

Conflicts of Interest:

The authors declare no conflict of interest.

Acknowledgments:

The authors are deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality of the paper.

Funding:

This research received no external funding.

Use of Generative-AI tools declaration:

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

References

1. Aslam, M., Albassam, M., & Jun, C. H. (2021). Economic determination of modified multiple dependent state sampling plan under some lifetime distributions. *International Journal of Quality & Reliability Management, 38*(7), 1755–1771.
2. Aslam, M., Azam, M., & Jun, C. H. (2015a). A control chart for exponential distribution using multiple dependent state sampling plan. *Quality & Quantity, 49*(2), 455–462.
3. Aslam, M., Nazir, A., & Jun, C. H. (2015b). A new attribute control chart using multiple dependent state sampling. *Transactions of the Institute of Measurement and Control, 37*(4), 569–576.
4. Aslam, M., Rao, G. S., & Albassam, M. (2019). Time truncated life tests using the generalized multiple dependent state sampling plans for various life distributions. *Statistical Quality Technologies, 9*(2), 153–182.
5. Aslam, M., Yen, C. H., Chang, C. H., & Jun, C. H. (2014). Multiple dependent state variable sampling plans with process loss consideration. *International Journal of Advanced Manufacturing Technology, 71*(5), 1337–1343.
6. Balamurali, S., & Jun, C. H. (2007). Multiple dependent state sampling plans for lot acceptance based on measurement data. *European Journal of Operational Research, 180*(3), 1221–1230.
7. Balamurali, S., Jeyadurga, P., & Usha, M. (2016). Designing of Bayesian multiple deferred state sampling plan based on Gamma–Poisson distribution. *American Journal of Mathematical and Management Sciences, 35*(1), 77–90.

8. Balamurali, S., Jeyadurga, P., & Usha, M. (2017a). Designing of multiple deferred state sampling plan for generalized inverted exponential distribution. *Sequential Analysis, 36*(1), 76–86.
9. Balamurali, S., Jeyadurga, P., & Usha, M. (2017b). Optimal designing of a multiple deferred state sampling plan for Weibull distributed lifetime assuring mean life. *American Journal of Mathematical and Management Sciences, 36*(2), 150–161.
10. Chhikara, R. S., & Folks, J. L. (1989). *The inverse Gaussian distribution*. Marcel Dekker.
11. Dodge, H. F. (1955). Chain sampling inspection plan. *Industrial Quality Control, 11*(3), 10–13.
12. Geetha, P., Rao, G. S., & Reddy, V. S. (2023). Design of multiple deferred state sampling inspection plan for variables. *Journal of Statistical Computation and Simulation, 93*(5), 859–878.
13. Govindaraju, K., & Subramani, K. (1993). Selection of multiple deferred (dependent) state sampling plans for given acceptable quality level and limiting quality level. *Journal of Applied Statistics, 20*(3), 423–428.
14. Jeyadurga, P., & Balamurali, S. (2020). Optimal designing of multiple deferred (dependent) state repetitive group sampling plan for variables inspection. *Communications in Statistics – Theory and Methods, 51*(3), 1–17.
15. Jilani, S. D., Rao, G. S., & Rao, A. V. (2024). Designing of multiple dependent state sampling plan for Type-II generalized half logistic distribution. *Research in Mathematics, 11*(1), 1–10.
16. Rao, G. S., Fulment, A. K., & Peter, J. K. (2021). Design of multiple dependent state sampling plan application for COVID-19 data using exponentiated Weibull distribution. *Complexity, 2021*, 1–10.
17. Rao, G. S., Rosaiah, K., & Naidu, C. R. (2020). Design of multiple-deferred state sampling plans for exponentiated half logistic distribution. *Cogent Mathematics & Statistics, 7*(1), 1857915.
18. Rao, G. S., Jilani, S. D., & Josephat, K. P. (2025). Designing of multiple dependent state sampling plan for Exponentiated Fréchet Distribution. *Research in Statistics, 3*(1), 1–8.
19. Soundararajan, V., & Vijayaraghavan, R. (1990). Construction and selection of multiple deferred (dependent) state sampling plan. *Journal of Applied Statistics, 17*(3), 397–409.
20. Subramani, K., & Haridoss, V. (2012). Development of multiple deferred state sampling plan based on minimum risks using the weighted Poisson distribution for given acceptable quality level and limiting quality level. *International Journal of Quality Engineering and Technology, 3*(2), 168–180.
21. Tahir, M. H., Cordeiro, G. M., Ali, S., Dey, S., & Manzoor, A. (2018). The inverted Nadarajah–Haghighi distribution: Estimation methods and applications. *Journal of Statistical Computation and Simulation, 88*(14), 2775–2798.
22. Vaerst, R. (1982). A procedure to construct multiple deferred state sampling plans. *Methods of Operations Research, 37*, 477–485.
23. Wortham, W. A., & Baker, R. C. (1976). Multiple deferred state sampling inspection. *International Journal of Production Research, 14*(6), 719–731.
24. Wu, C. W., Lee, A. H. I., & Chen, Y. W. (2016). A novel lot sentencing method by variables inspection considering multiple dependent states. *Quality and Reliability Engineering International, 32*(3), 985–999.
25. Wu, C. W., Liu, S. W., & Lee, A. H. I. (2015). Design and construction of variables multiple dependent state sampling plan based on process yield. *European Journal of Industrial Engineering, 9*(6), 819–838.
26. Yen, C. H., Chang, C. H., & Lee, C. C. (2023). A new multiple dependent state sampling plan based on one-sided process capability indices. *International Journal of Advanced Manufacturing Technology*, 126(7), 3297–3309.
27. Zang, Z., Xu, X., Yang, C., & Chen, A. (2018). A closed-form estimation of the travel time percentile function for characterizing travel time reliability. *Transportation Research Part B: Methodological, 118*, 228–247.