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Research article

A New Transformation to Reduce Skewness in Data with Bounded Support

Mahmoud A. Eltehiwy[1,*], **Noura A. Taha AbuEl-magd**[1]

[1] Department of Statistics, Faculty of Politics and Economics, Beni Suef University, Beni Suef 62511, Egypt; Mahmoud.abdelmone@eps.bsu.edu.eg; Noura.shon@eps.bsu.edu.eg

* **Correspondence:** Mahmoud.abdelmone@eps.bsu.edu.eg

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ABSTRACT

Normality is a critical assumption for many statistical analyses, yet real-world data often deviate from this requirement. To address this, statisticians may employ robust methods or apply transformations to achieve approximate normality. This study proposes a novel transformation for data from bounded-support distributions, such as the beta family, to induce symmetry or near-symmetry. The method utilizes a data-driven parameter, estimated through a simple and efficient procedure. Examples demonstrate the transformation's effectiveness, and its straightforward implementation allows easy reversion to the original data scale. We recommend this approach for statistical analyses requiring normality, even for datasets that are approximately normal, due to their versatility and robustness.

1. Introduction

In statistical analysis, much of the theory is developed under the assumption of normality. However, in practice, researchers often encounter data that deviates from this assumption and data follows a skewed distribution [6, 3].This assumption is crucial for the validity of many parametric methods, including analysis of variance (ANOVA) and linear mixed models, as well as for the efficiency of some non-parametric techniques [17]. When the assumption of normality is violated, data transformations are often employed to approximate or achieve a normal distribution. Such transformations offer the advantage of preserving all observations while potentially reducing skewness and enhancing the interpretability of results [5, 16].

Transformations are particularly useful in studies with repeated measures or multiple trials, where normality of the data significantly affects inferential power [1]. Several studies have demonstrated that applying suitable transformations can reveal statistically significant effects that may remain undetected under untransformed conditions [12]. Kang et al. [11] introduced a new family of density functions to model skewness. Their approach employs a transformation that incorporates a shape parameter, functioning analogously to traditional location and scale adjustments. Parra-Frutos and Molera [19] proposed transformation of the Welch statistic aims to reduce both skewness and kurtosis effects on Type I error rates. It builds on the Cornish-Fisher expansion, incorporating additional terms to address Kurtosis. The modified t variable is derived by adjusting the usual statistics, ensuring high-order biases are removed. Hamasha et al. [7] introduced the Ultra-fine transformation method, which can convert any data into a standard normal distribution, regardless of its complexity or original distribution shape. This is a significant advancement over traditional methods that often have limitations on the types of data they can handle. Huang et al. [9] developed a novel Newton-type algorithm to implement the log, Box-Cox and log-sinh transformations for precipitation data; The algorithm explicitly accounts for the skewness and censoring characteristics of precipitation data by the likelihood function. The computational efficiency and algorithm convergence are illustrated for the log, Box-Cox and log-sinh transformations. Lavrač and Turk [13] proposed modified power transformation method effectively reduces skewness in non-normal process capability estimation compared to other methods. Liu et al. [15] presented a novel and efficient unified framework for static and dynamic reliability analysis of engineering structures by integrating the dual power transformation, Yeo-Johnson transformation, and the fourth-moment method.

Among transformation methods, the Box-Cox power transformation stands out for its flexibility and empirical validation across disciplines [18]. Developed by [4], this parametric technique employs a single parameter (λ) to adjust data skewness, ensuring adherence to linear model assumptions [20]. The transformation modifies Tukey's (1977) [21] power family to accommodate discontinuities at $\lambda = 0$, enabling its application to positive-valued datasets [14].

Let $\mathbf{y} = (y_1, y_2, \ldots, y_n)'$ be the data on which the Box-Cox transformation is to be applied. Box and Cox [4] introduced their power transformation as follows:

$$y_i^{(\lambda)} = \begin{cases} \dfrac{y_i^{\lambda} - 1}{\lambda}, & \text{if } \lambda \neq 0, \\ \ln(y_i), & \text{if } \lambda = 0, \end{cases} \tag{1.1}$$

such that, for unknown λ,

$$\mathbf{y}^{(\lambda)} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where $\mathbf{y}^{(\lambda)}$ is the λ-transformed data, X is the design matrix (possible covariates of interest), $\boldsymbol{\beta}$ is the set of parameters associated with the λ-transformed data, and $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ is the error term. Since the aim of Equation (1.1) is that

$$\mathbf{y}^{(\lambda)} \sim N(X\boldsymbol{\beta}, \sigma^2 I_n),$$

then $\boldsymbol{\varepsilon} \sim N(0, \sigma^2)$. The goal of this transformation is to achieve a linear model with normally distributed errors. Note that the transformation in Equation (1.1) is only valid for $y > 0$, and modifications have been made when zero or negative observations are present [22]. Maximum likelihood estimation under the normality assumption is typically used to determine the optimal value of λ [4]. Despite its utility, the Box-Cox transformation is limited to positive data and requires modifications for datasets containing non-positive values [22]. Alternative approaches, such as those proposed by [8], focus on achieving symmetry or near-symmetry in the transformed data by estimating λ based on quantile matching. However, these methods may not be suitable for all types of distributions, particularly those with bounded support. The estimate of λ is the solution of

$$y_{(1-q)}^{\lambda} - M^{\lambda} = M^{\lambda} - y_q^{\lambda},$$

where y_q and $y_{(1-q)}$ are the lower and upper q^{th} quantiles ($0 < q < \frac{1}{2}$), and M is the median.

In this paper, we propose a new transformation specifically designed for data arising from distributions with bounded support, particularly those within the beta distribution family. We assume the support is the interval $[a, b]$, with both endpoints known or estimable. Assuming that the parent distribution is a member of the four-parameter beta $(p, q; a, b)$ family, then with both a and b known, the transformation $t(x) = \frac{x-a}{b-a}$ would yield a standard beta (p, q) variable, where $p > 0$ and $q > 0$ determine the distribution's shape and degree of skewness.

It has been known that if X has a beta (p, q) distribution, the transformation $Y = \ln\left(\frac{X}{1-X}\right)$ yields a variable nearer to symmetry than X (see [10]). Clearly, if X is symmetric about $1/2$, then Y is symmetric about zero. Moreover, the transformed data will have a bell-shaped distribution that approaches normality as p becomes large. For skewed X, the transformed variable Y is also skewed.

We propose a new transformation of the form

$$Y = \ln\left(\frac{X^{\lambda}}{1-X}\right), \qquad \lambda > 0, \tag{1.2}$$

where the parameter λ is chosen such that Y is either symmetric or its skewness is very small.

This paper introduces a novel transformation designed to induce symmetry in data drawn from bounded distributions, particularly the four-parameter beta family. Unlike existing methods, the proposed technique incorporates an estimable parameter to optimize symmetry, leveraging the beta distribution's properties to achieve near-normality through a computationally straightforward process.

In Section 2, we determine the value of λ that satisfies our symmetry condition. We then introduce our estimator of λ and discuss some of its properties. Section 3 presents the results of a simulation study comparing the exact forms of the distributions of X with the empirical distribution of Y evaluated from generated data and the estimate of λ. It also contains the results of a Monte Carlo study evaluating the performance of the method in moderate sample sizes, by comparing the skewness of data before and after transformation. Section 4 provides the concluding remarks.

2. Estimation of λ

Let X be a random variable with a standard beta (p, q) distribution. We assume that $p \leq q$. This assumption does not cause any loss of generality, since if $p > q$ the transformation $X' = (1 - X)$ yields a variable satisfying our assumption.

Consider the transformation given by Equation (1.2). Our objective here is to find a value (or values) of λ that produces a symmetric or "near symmetric" variable Y. Since the third central moment of a variable is often used as a measure of the degree of asymmetry of its distribution, we will use this to evaluate λ such that the third central moment of Y is zero or near zero.

Let μ_3 denote the third central moment of Y. Clearly, μ_3 and λ are functions of p and q. As a measure of asymmetry, the coefficient of skewness is given by $c_s = \frac{\mu_3}{\mu_2^{3/2}}$.

For our subsequent discussion, we need the following results.

Lemma 2.1. *Let Y be defined as in Equation (1.2). Then its third central moment, μ_3, is given by*

$$\mu_3 = \lambda^3\Psi^{(2)}(p) - \Psi^{(2)}(q) + (1-\lambda)^3\Psi^{(2)}(p+q), \tag{2.1}$$

where $\Psi^{(2)}(s)$ is the second derivative of the digamma function $\Psi(s)$.

Proof. Let $\phi(t)$ be the cumulant generating function of Y. It is easily obtained that

$$\phi(t) = [\ln\Gamma(p+\lambda t) - \ln\Gamma(p)] + [\ln\Gamma(q-t) - \ln\Gamma(q)] - [\ln\Gamma(p+q-(1-\lambda)t) - \ln\Gamma(p+q)]. \tag{2.2}$$

Since $\mu_3 = \phi^{(3)}(t)\big|_{t=0}$, it follows that μ_3 is as given in Equation (2.1). □

Lemma 2.2. *Viewing μ_3 as a third-degree polynomial in λ, the equation $\mu_3 = 0$ has only one positive root λ_1 in the interval* $(0,1)$.

Proof. The equation $\mu_3 = 0$ can be written as

$$\mu_3 = \lambda^3 + 3C\lambda^2 - 3C\lambda - (1-d) = 0, \tag{2.3}$$

where

$$C = -\frac{\Psi^{(2)}(p+q)}{\Psi^{(2)}(p+q) - \Psi^{(2)}(p)} \quad \text{and} \quad d = \frac{\Psi^{(2)}(q) - \Psi^{(2)}(p)}{\Psi^{(2)}(p+q) - \Psi^{(2)}(p)}.$$

Observing that $\Psi^{(2)}(s) < 0$ and is increasing for all $s > 0$, it follows that $C > 0$ and $0 \le d < 1$. For $p \le q$, we consider two cases: □

Case 1. If $p = q$, then $d = 0$ and Equation (2.3) becomes

$$\lambda^3 + 3C\lambda^2 - 3C\lambda - 1 = 0.$$

Using elementary analytical methods, it is easy to show that this equation has one real positive root $\lambda_1 = 1$, and either two real negative roots when $C \ge \frac{1}{3}$, or two complex conjugate roots when $C < \frac{1}{3}$.

Case 2. If $p < q$, which implies that $0 < d < 1$, then from Equation (2.3) we obtain:

$$\text{as } \lambda = 0, \quad \mu_3 = d - 1 < 0, \qquad \text{and as } \lambda = 1, \quad \mu_3 = d > 0.$$

Therefore, Equation (2.3) has at least one real positive root in $(0,1)$. Assume that the value of this root is $e > 0$, and that the other two roots are λ_2 and λ_3. It is easy to show that

$$\lambda_2 + \lambda_3 = -3C - e < 0, \qquad \text{and} \qquad \lambda_2\lambda_3 = \frac{1-d}{e} > 0.$$

Only two alternatives are possible. The first is when λ_2 and λ_3 are both real and negative. The second is when they are complex and conjugate. Therefore, in all cases, the equation has only one real root between zero and one.

This last result, in addition to extensive examination of the behavior of the functions $\Psi^{(2)}(s)$ and $s^3\Psi^{(2)}(s)$, suggests using the simple function

$$\lambda(p, q) = \frac{p}{q}$$

as an approximation of λ (see [2]). Using this value of λ, we have

$$\mu_3 = \frac{1}{q^3}[p^3\Psi^{(2)}(p) - q^3\Psi^{(2)}(q) + (q-p)^3\Psi^{(2)}(p+q)]. \tag{2.4}$$

The value of μ_3 is exactly zero for $p = q$. Table 1 compares the values of the coefficient of skewness, C_S, of $X \sim \text{Beta}(p, q)$ and the corresponding C_S of Y (the transformed variable) for several choices of p and q. The last column of Table 1 gives the ratio of $C_S(Y)$ to $C_S(X)$. It is clear from the Table 1 that

Table 1. Effect of the proposed transformation on skewness.

Shape parameter p	**Shape parameter** q	**Skewness Before** $C_S(X)$	**Skewness After** $C_S(Y)$	**Ratio** $\frac{C_S(Y)}{C_S(X)}$
0.25	0.50	0.664	0.031	0.046
0.50	0.75	0.365	0.075	0.205
0.50	2.00	1.229	0.068	0.055
1.00	3.00	0.862	0.136	0.158
1.50	6.00	0.931	0.181	0.194
2.00	4.00	0.471	0.123	0.261
2.00	8.00	0.816	0.173	0.211
2.50	6.50	0.570	0.130	0.228
2.50	10.0	0.758	0.154	0.203
3.00	5.00	0.312	0.083	0.267
4.00	8.00	0.384	0.115	0.300
5.00	10.0	0.339	0.105	0.308

the transformation reduces the skewness of the original distribution. The greatest reduction occurred when $p = 0.25$ and $q = 0.5$. The least reduction was for $p = 5$ and $q = 10$.

Let us now assume that we have data from a beta model with unknown parameters p and q. If it is our desire to transform the data to attain symmetry or near symmetry, an estimate of λ is needed. Since $\bar{X}$ is the moment estimate of $\frac{p}{p+q}$, then by writing

$$\frac{p}{p+q} = \frac{\lambda}{\lambda + 1},$$

it follows that the moment estimate of λ is

$$\hat{\lambda} = \frac{\bar{X}}{1 - \bar{X}}.$$

It is easily seen that $\hat{\lambda}$ converges almost surely to $\frac{p}{q}$. This optimal asymptotic property and simplicity of $\hat{\lambda}$ are appealing. It remains to investigate the performance of the transformation using equation (1.2). The results of this investigation are presented in the next section.

3. Simulation Methodology and Results

Having established the theoretical foundation for the transformation, we now turn to its empirical performance. The following simulations assess whether the practical application of the transformation—using the estimated $\hat{\lambda}$ on finite samples—fulfills the theoretical promise of inducing symmetry.

To evaluate the performance of the proposed transformation, two criteria have been examined. First, a comparison of the shape of the empirical distribution of the transformed data with the shape of the original distribution is made. Second, a comparison of the skewness of data in moderate sample sizes before and after transformation is conducted.

Because λ is estimated from the data, a Monte Carlo simulation is used to study the performance of the proposed transformation in different settings. For the first criterion, six distributions from the beta (p, q) family were chosen. Three of the chosen distributions are symmetric, while the others are skewed. For each of the six distributions, 20,000 observations were generated and λ was estimated.

The transformation $Y = \ln\left(\frac{X^{\lambda}}{1-X}\right)$ was applied to the generated data after replacing λ by its moment estimate, $\hat{\lambda} = \frac{\bar{X}}{1-\bar{X}}$. The histograms of the original data and those of the transformed data were plotted next to the histogram of the original distribution.

Figure 1 shows the histograms of each of the three symmetric beta distributions along with the corresponding histograms of the transformed data. Examining Figure 1, one observes that the symmetry of the distributions was not violated by the transformation. Moreover, the shapes of the transformed variables are now very close to the popular bell shape.

Figure 2 shows the histograms for each of the three skewed data distributions along with the corresponding histograms for the transformed data. The change of the shapes to symmetric bell-shaped ones is quite obvious. It is clear from Figure 1 and Figure 2 that the new transformation is highly effective in transforming various shapes of distributions to near symmetry. The skewed distributions and the symmetric ones are all transformed into bell-shaped distributions.

For the second criterion, we conducted a comprehensive Monte Carlo simulation to quantitatively evaluate the skewness reduction across various sample sizes and distribution shapes. We generated 10,000 random samples for each of three sample sizes ($n = 30, 50, 100$) from three skewed beta distributions: Beta$(0.5, 2)$, Beta$(1.5, 6)$, and Beta$(2.5, 8)$. For each sample, the transformation was applied using the estimated $\hat{\lambda}$, and the sample skewness was calculated before and after transformation.

The results, summarized in Table 2, demonstrate a substantial reduction in skewness across all scenarios. For instance, with a sample size of $n = 30$ drawn from the Beta distribution $(2.5, 8)$, the mean and median coefficients of skewness for the original data were found to be 0.548 and 0.528, while after transformation, they were 0.119 and 0.114, respectively, indicating an impressive reduction of 78.28% and 78.41%. This consistent performance is visually confirmed in Figure 3, where the box-plots show a dramatic downward shift and compression in the distribution of skewness values after transformation.

The method proved to be highly effective even for the highly skewed Beta $(0.5, 2)$ distribution, achieving over 93% reduction in mean skewness for $n = 30$. Furthermore, the results indicate that the transformation's efficacy is robust across the different sample sizes studied, with consistent percentage reductions observed for $n = 30, 50$, and 100. This confirms the practical utility of the transformation for the moderate sample sizes commonly encountered in applied research. The simulation study suggests that the new transformation reduces the skewness of the data.

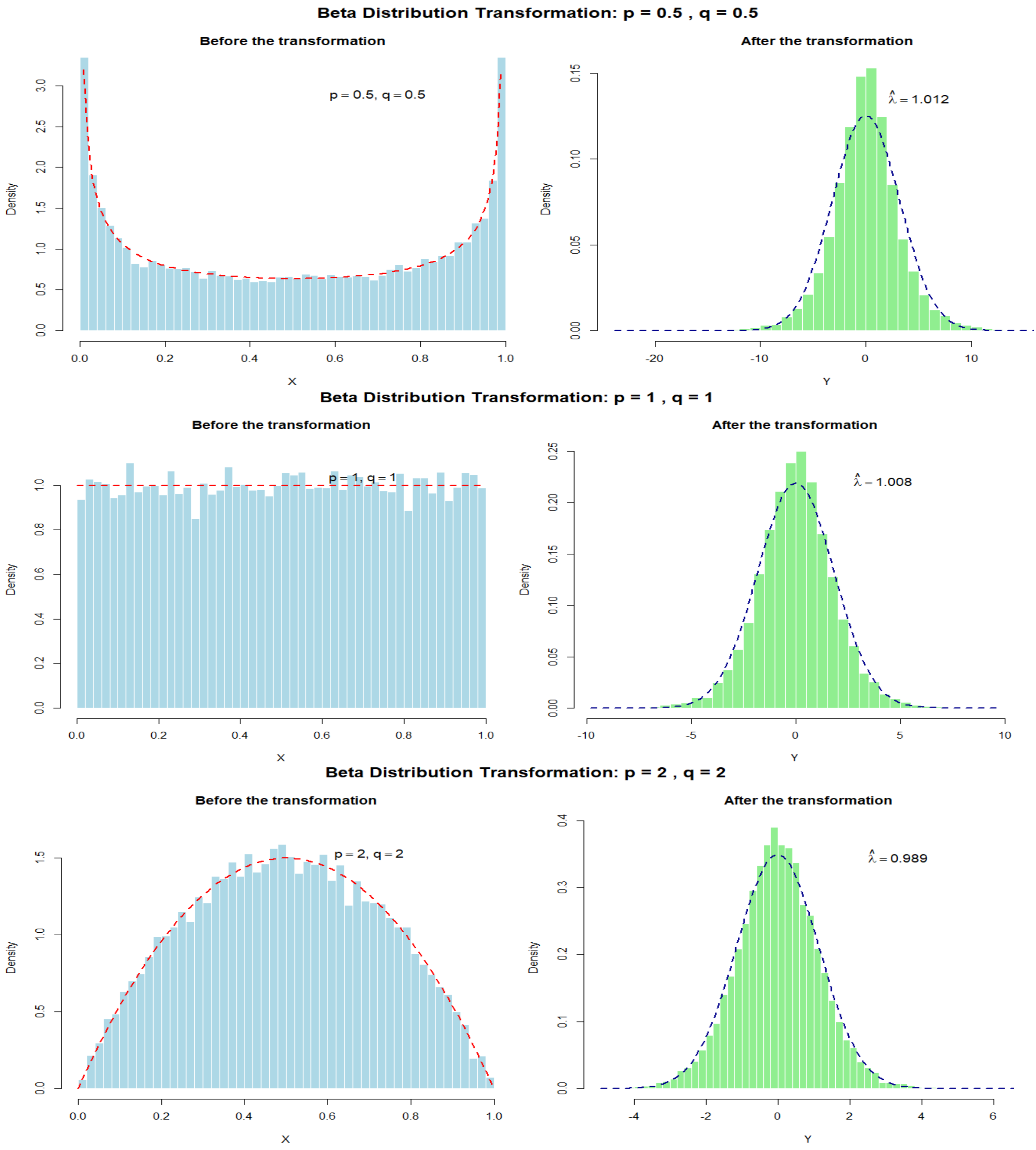

Figure 1. Histograms for a symmetric beta distribution and its transformed counterpart using the proposed method, with shape parameters $(p, q) = (0.5, 0.5)$, $(1, 1)$, and $(2, 2)$. Sample size $n = 20{,}000$.

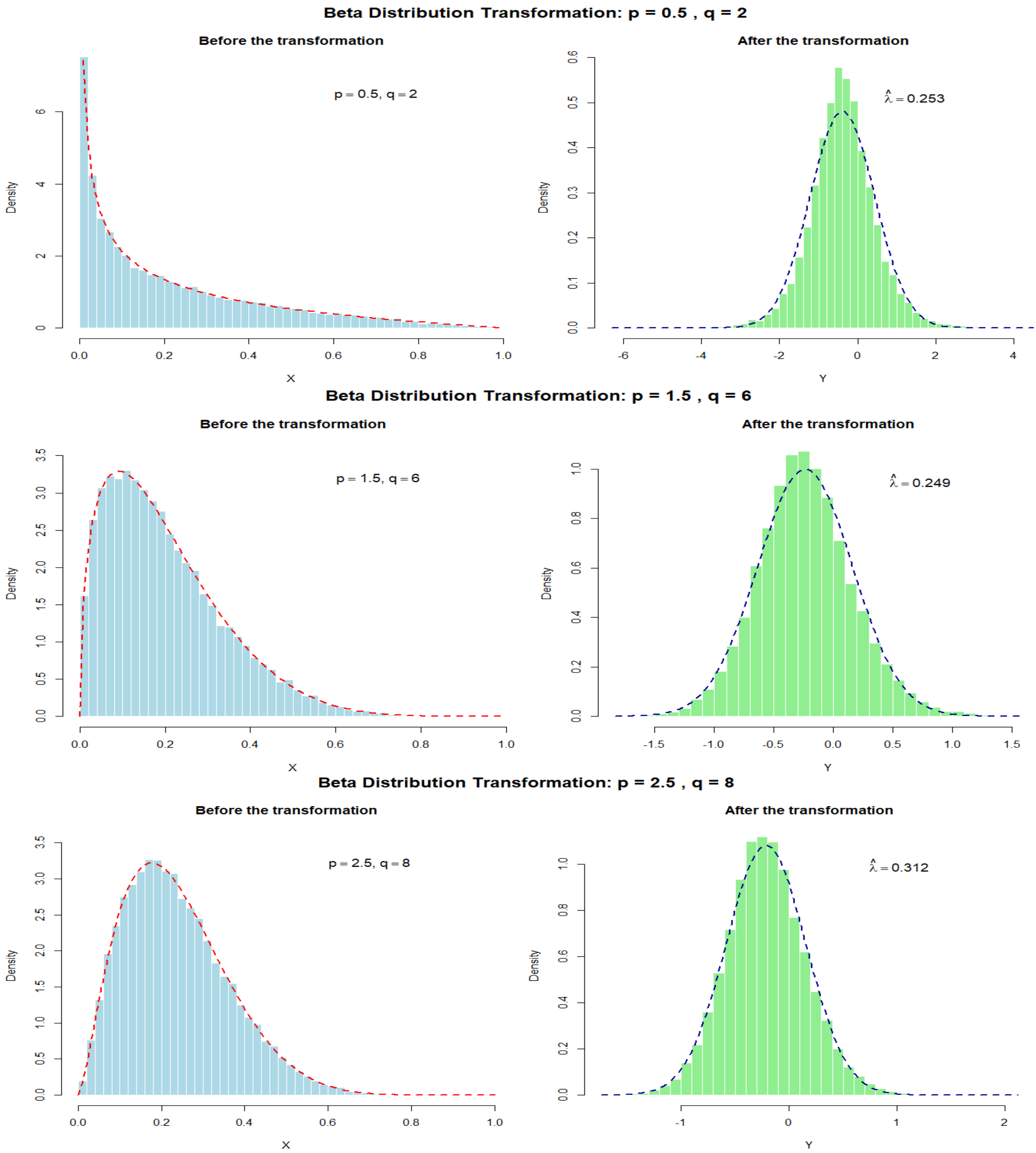

Figure 2. Histograms for skewed beta distribution and its transformed counterpart using the proposed method, with shape parameters $(p, q) = (0.5, 2)$, $(1.5, 6)$, and $(2.5, 8)$. Sample size $n = 20{,}000$.

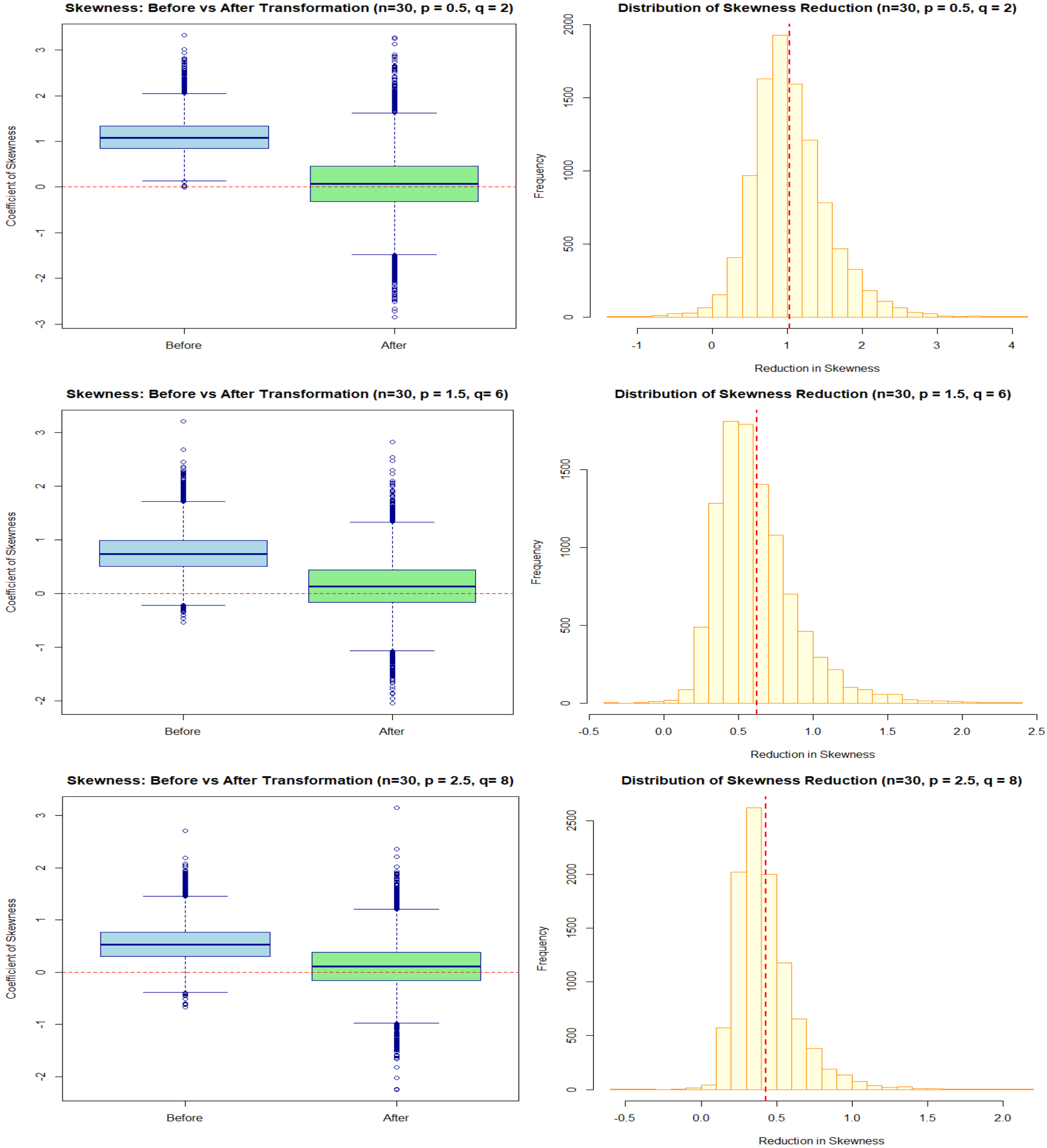

Figure 3. Box-plots for skewed beta distribution and distribution of skewness reduction with shape parameters , with shape parameters $(p, q) = (0.5, 2)$, $(1.5, 6)$, and $(2.5, 8)$, $n = 30$.

Table 2. Monte Carlo Simulation Results: Reduction in skewness before and after transformation.

	$p = 0.5, q = 2$					
n	Skewness before (mean)	Skewness after (mean)	Reduction (%)	Skewness before (median)	Skewness after (median)	Reduction (%)
30	1.111	0.075	1.035 (93.16%)	1.081	0.070	1.012 (93.62%)
50	1.169	0.077	1.092 (93.41%)	1.148	0.069	1.078 (93.90%)
100	1.211	0.082	1.129 (93.23%)	1.200	0.079	1.122 (93.5%)
n	$p = 1.5, q = 6$					
30	0.765	0.142	0.623 (81.43%)	0.741	0.134	0.607 (81.92%)
50	0.826	0.152	0.674 (81.6%)	0.807	0.148	0.660 (81.78%)
100	0.873	0.164	0.709 (81.21%)	0.858	0.156	0.702 (81.82%)
n	$p = 2.5, q = 8$					
30	0.548	0.119	0.429 (78.28%)	0.528	0.114	0.414 (78.41%)
50	0.594	0.129	0.466 (78.45%)	0.578	0.127	0.452 (78.2%)
100	0.630	0.137	0.493 (78.25%)	0.621	0.134	0.487 (78.42%)

4. Concluding Remarks

This paper introduced a novel transformation, $Y = \ln\left(\frac{X^\lambda}{1-X}\right)$, designed to induce symmetry in data arising from bounded distributions. The transformation is particularly well-suited for data that can be modelled by the Beta distribution family, where the variable X has support on the interval $(0, 1)$. The core of the method involves estimating a single parameter, λ, which optimally balances the influence of the two tails of the distribution to achieve near-symmetry in the transformed variable Y.

We demonstrated that the equation for the third central moment of Y has a unique root in the interval $(0, 1)$, guaranteeing the existence of an optimal λ. Rather than relying on complex numerical solutions, we proposed a simple, consistent, and computationally efficient moment estimator, $\hat{\lambda} = \frac{\bar{X}}{1-\bar{X}}$. The efficacy of the transformation was validated through a comprehensive simulation study. The results visually confirmed that the method successfully reshapes both symmetric and skewed Beta distributions into approximately symmetric, bell-shaped curves. Furthermore, a quantitative analysis demonstrated a substantial reduction in skewness, with an average reduction of over 80% in a representative scenario, confirming its practical utility even for moderate sample sizes.

While the proposed transformation demonstrates significant promise, it is essential to acknowledge its limitations and areas for future investigation. The most significant limitation, as rightly noted, is that the transformation is primarily designed for distributions with support bounded between 0 and 1. Its theoretical derivation is rooted in the properties of the Beta distribution. Therefore, its direct application is most appropriate for data that naturally fall within this unit interval (e.g., proportions, rates, percentages scaled to [0,1]) and are suspected to follow a Beta-like distribution. Applying it to unbounded data or data with different support is not theoretically justified by this work.

Although we focused on the standard $[0, 1]$ interval, the method can be easily extended to any bounded interval $[a, b]$ through a preliminary linear transformation. By defining $Z = \frac{X-a}{b-a}$, the variable Z will lie in $[0, 1]$, and our proposed transformation can then be applied to Z. Future work could explicitly explore the performance of the method on such generalized bounded data.

The simulations in this paper were confined to the Beta family. The performance of the transformation when applied to data from other bounded, non-Beta distributions (e.g., Kumaraswamy, truncated normal) remains an open question and a valuable direction for future research.

In conclusion, we have presented a theoretically grounded, simple, and effective transformation for reducing skewness in bounded data. Its ease of implementation and interpretation makes it a valuable addition to the tool-kit of practitioners working with proportional data and related constructions. We encourage its use in statistical analyses where achieving symmetry is a crucial step, provided the data's bounded nature aligns with the method's assumptions.

Authors' Contributions

All authors have worked equally to write and review the manuscript.

Data Availability Statement

The data that supports the findings of this study are available within the article.

Conflicts of Interest

The authors declare no conflict of interest.

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