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Research article

Development and analysis of a new trigonometric unit distribution

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ABSTRACT

In this paper, we introduce a novel trigonometric unit distribution that generalizes the uniform distribution through a single shape parameter. We derive its main properties, including closed-form expressions for the cumulative distribution function and the mean. We then study the associated statistical model and assess its inferential performance using maximum likelihood estimation. A simulation study confirms the effectiveness of the proposed estimation procedure. Applications to two real data sets show that the new model provides greater flexibility and achieves competitive goodness-of-fit compared with traditional alternatives.

1. Introduction

In recent years, the development of unit (probability) distributions has gained significant traction due to their versatility in modeling real-world phenomena constrained within the unit interval, $[0, 1]$. From recovery rates in medicine to proportion indices in economics and environmental science, the demand for flexible models that extend beyond the classical models is ever-increasing. In particular, while the uniform distribution serves as the fundamental baseline for such data, it often fails to capture the underlying skewness or kurtosis present in empirical observations. Key examples of unit distributions include the truncated

Weibull distribution (TWD) [13], unit-Weibull (UW) distribution [8], unit-exponentiated half-logistic distribution [9], unit inverse exponentiated Lomax distribution [10], unit–power Burr X distribution [11], new unit-lindley model [12], truncated new X-Lindley distribution (TNXLD) [7], truncated moment exponential distribution (TMED) [5], and truncated power Lomax distribution (TPLD) [6]. For a comprehensive overview of this topic, we refer the reader to the survey by [2], which contains an extensive list of related work (185 references).

To address the limitations of the uniform distribution, we introduce the new trigonometric unit (NTU) distribution. It is characterized by a probability density function (PDF) that incorporates a sine-based trigonometric component, governed by a single shape parameter $\alpha \in [-1, 1]$. The NTU distribution offers several distinct advantages: it maintains a parsimonious structure while providing a wide range of PDF forms, from concave to convex, depending on the sign and magnitude of the parameter. A defining feature of the NTU distribution is its relationship with the standard uniform distribution; specifically, when $\alpha = 0$, the model recovers the uniform case. This makes the NTU distribution a natural one-parameter generalization, offering practitioners a robust alternative for data fitting and statistical inference.

In the following sections, we derive the fundamental properties of the NTU distribution, including its cumulative distribution function (CDF) and moments (Sections 2 and 3). Furthermore, we explore its practical utility through a series of parameter estimation simulations and applications to real-world datasets (Sections 4 and 5). A conclusion is given in Section 6.

2. The New Unit Distribution

The theorem below presents a valid PDF of a unit distribution at the basis of the NTU distribution.

Theorem 2.1. *The following function is a valid PDF with support* $[0, 1]$*:*

$$f(x) = \frac{1}{1 + 2\alpha/\pi} (1 + \alpha \sin(\pi x)), \quad x \in [0, 1],$$

with $\alpha \in [-1, 1]$*, which is completed by* $f(x) = 0$ *for any* $x \notin [0, 1]$.

Proof. For any $x \in [0, 1]$, since $\alpha \in [-1, 1]$ and $\sin(\pi x) \in [0, 1]$, we have

$$f(x) = \frac{1}{1 + 2\alpha/\pi} (1 + \alpha \sin(\pi x)) \geq \frac{1}{1 + 2/\pi} (1 - \sin(\pi x)) \geq 0.$$

Therefore, for any $x \in \mathbb{R}$, we have $f(x) \geq 0$. It is clear that f is continuous over $\mathbb{R} \backslash \{0, 1\}$. Furthermore, we have

$$\begin{aligned}
\int_{-\infty}^{+\infty} f(x) dx &= \int_0^1 \frac{1}{1 + 2\alpha/\pi} (1 + \alpha \sin(\pi x))\, dx \\
&= \frac{1}{1 + 2\alpha/\pi} \int_0^1 (1 + \alpha \sin(\pi x))\, dx \\
&= \frac{1}{1 + 2\alpha/\pi} \left[x - \alpha \frac{\cos(\pi x)}{\pi} \right]_0^1 \\
&= \frac{1}{1 + 2\alpha/\pi} \left(1 + \alpha \frac{1}{\pi} - 0 + \alpha \frac{1}{\pi} \right) = 1.
\end{aligned}$$

We conclude that f is a valid PDF with support $[0, 1]$, completing the proof. □

We define the unit distribution with PDF f as the NTU distribution. For $\alpha = 0$, the NTU distribution reduces to the uniform distribution over the interval $[0, 1]$. Consequently, the parameter α allows the NTU distribution to be interpreted as a one-parameter generalization of the uniform case. In a sense, this distribution introduces a controlled periodic deformation of the uniform distribution. The parameter α governs the strength of the sinusoidal modulation while preserving normalization.

Figure 2 plots this PDF for several values of α.

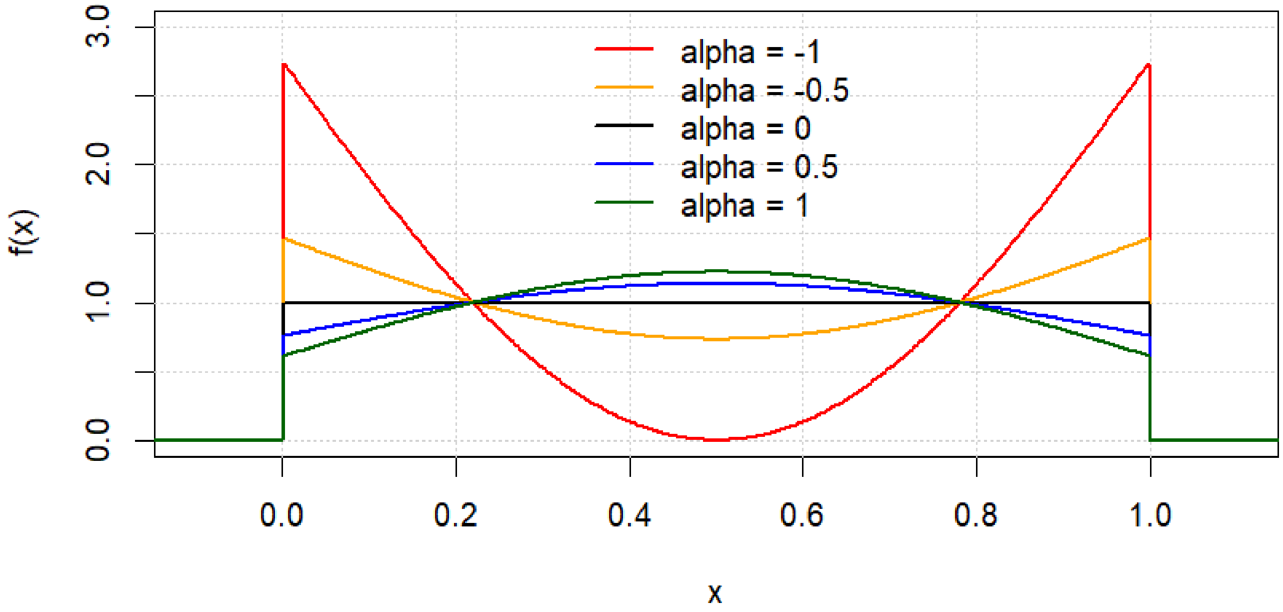

Figure 1. PDF of the NTU distribution for various values of α.

Various concave and convex forms are observed, showing the versatility of the NTU distribution. We also observe that the PDF is symmetrical with respect to $x = 1/2$.

3. Some Basic Properties

The CDF associated with the NTU distribution is presented in the theorem below.

Theorem 3.1. *The CDF associated with the NTU distribution is given by*

$$F(x) = \frac{1}{1 + 2\alpha/\pi}\left(x + \alpha\frac{1 - \cos(\pi x)}{\pi}\right), \qquad x \in [0, 1],$$

with $\alpha \in [-1, 1]$, *which is completed by* $F(x) = 0$ *for any* $x < 0$, *and* $F(x) = 1$ *for any* $x > 1$.

Proof. For any $x \in \mathbb{R}$, we have $F(x) = \int_{-\infty}^{x} f(t)dt$, where f denotes the PDF associated with the NTU distribution. It is clear that $F(x) = 0$ for any $x < 0$, and $F(x) = 1$ for any $x > 1$. For any $x \in [0, 1]$, we have

$$\begin{aligned}
F(x) &= \int_{-\infty}^{x} f(t)dt = \int_{0}^{x} \frac{1}{1 + 2\alpha/\pi}(1 + \alpha \sin(\pi t))\, dt \\
&= \frac{1}{1 + 2\alpha/\pi}\int_{0}^{x} (1 + \alpha \sin(\pi t))\, dt \\
&= \frac{1}{1 + 2\alpha/\pi}\left[t - \alpha\frac{\cos(\pi t)}{\pi}\right]_{0}^{x} \\
&= \frac{1}{1 + 2\alpha/\pi}\left(x - \alpha\frac{\cos(\pi x)}{\pi} - 0 + \alpha\frac{1}{\pi}\right) \\
&= \frac{1}{1 + 2\alpha/\pi}\left(x + \alpha\frac{1 - \cos(\pi x)}{\pi}\right).
\end{aligned}$$

This concludes the proof. □

Figure 2 plots this CDF for several values of α.

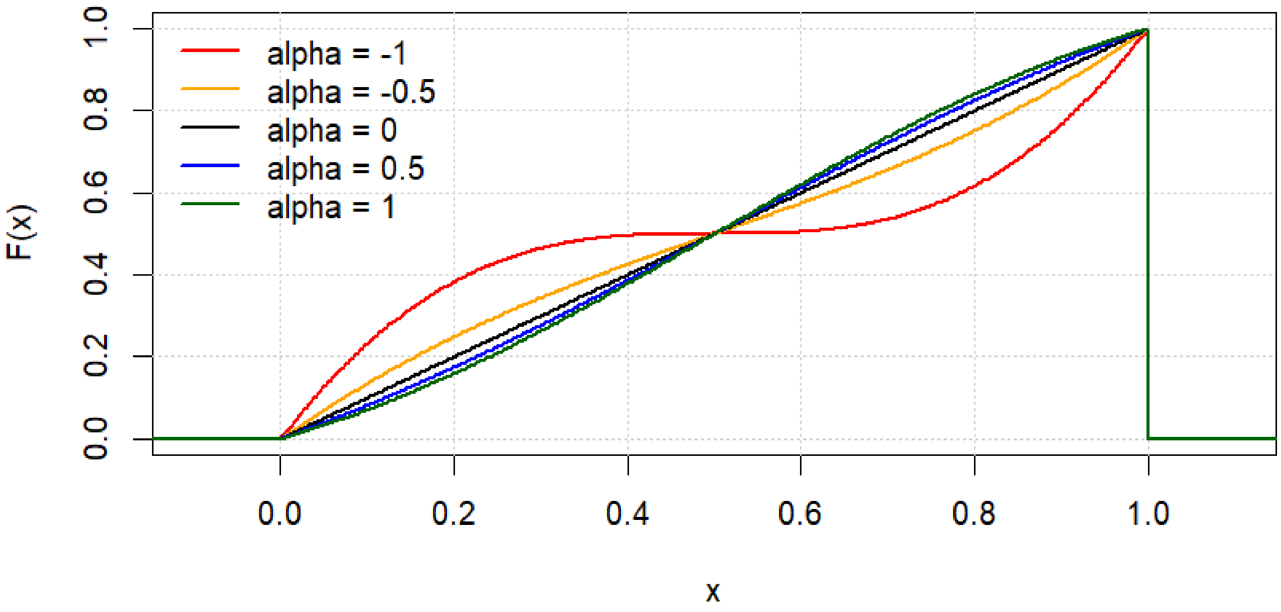

Figure 2. CDF of the NTU distribution for various values of α.

Various concave and convex forms are observed, showing the versatility of the NTU distribution.

The expression of the CDF does not admit a closed-form expression for the associated quantile function, although it can still be evaluated numerically.

The mean associated with the NTU distribution is presented in the theorem below.

Theorem 3.2. *The mean associated with the NTU distribution is given by*

$$\mu = \frac{1}{2}$$

Proof. We have $\mu = \int_{-\infty}^{+\infty} xf(x)dx$, where f denotes the PDF associated with the NTU distribution. Using standard integral formulas and an integration by parts, we get

$$\begin{aligned}
\mu &= \int_{-\infty}^{+\infty} xf(x)dx = \int_0^1 x\frac{1}{1+2\alpha/\pi}\left(1+\alpha\sin(\pi x)\right)dx \\
&= \frac{1}{1+2\alpha/\pi}\int_0^1 x\left(1+\alpha\sin(\pi x)\right)dx \\
&= \frac{1}{1+2\alpha/\pi}\left(\int_0^1 xdx + \alpha\int_0^1 x\sin(\pi x)dx\right) \\
&= \frac{1}{1+2\alpha/\pi}\left(\left[\frac{1}{2}x^2\right]_0^1 + \alpha\left(\left[-x\frac{1}{\pi}\cos(\pi x)\right]_0^1 - \int_0^1 -\frac{1}{\pi}\cos(\pi x)dx\right)\right) \\
&= \frac{1}{1+2\alpha/\pi}\left(\frac{1}{2}+\frac{\alpha}{\pi}+\alpha\left[\frac{1}{\pi^2}\sin(\pi x)\right]_0^1\right) \\
&= \frac{1}{1+2\alpha/\pi}\left(\frac{1}{2}+\frac{\alpha}{\pi}+0\right) = \frac{1}{2}.
\end{aligned}$$

This concludes the proof. □

Some other properties of the NTU distribution are briefly presented below.

- Using the integral formulas

$$\int_0^1 x^2dx = \frac{1}{3}, \quad \int_0^1 x^2\sin(\pi x)dx = \frac{1}{\pi}-\frac{4}{\pi^3},$$

  we can prove that the variance of the NTU distribution is

$$\sigma^2 = \frac{1}{1+2\alpha/\pi}\left(\frac{1}{3}+\alpha\left(\frac{1}{\pi}-\frac{4}{\pi^3}\right)\right)-\frac{1}{4}.$$

- Using the series decomposition of the sine function, for any $x \in [0,1]$, we can write

$$f(x) = \frac{1}{1+2\alpha/\pi}\left(1+\alpha\sum_{n=0}^{+\infty}(-1)^n\frac{\pi^{2n+1}x^{2n+1}}{(2n+1)!}\right).$$

  Based on it, for any integer k, the moment of order k associated with the NTU distribution is given by

$$\mu_k = \int_{-\infty}^{+\infty} x^k f(x)dx = \frac{1}{1+2\alpha/\pi}\left(\frac{1}{k+1}+\alpha\sum_{n=0}^{+\infty}(-1)^n\frac{\pi^{2n+1}}{(k+2n+2)(2n+1)!}\right).$$

  To the best of our knowledge, there is no closed-form expression for this moment.

- For any continuous baseline CDF G, the composite function $F \circ G$ is itself a valid CDF. This allows the NTU distribution to serve as a generator for a wide array of new distributions, a methodology explored extensively in [4]. For example, let us consider the CDF of the exponential distribution given by

  $$G(x) = 1 - e^{-\lambda x}, \quad x > 0,$$

  where $\lambda > 0$, which is completed by $G(x) = 0$ for any $x \leq 0$. Then, we define a new lifetime distribution by the following CDF:

  $$K(x) = (F \circ G)(x) = \frac{1}{1 + 2\alpha/\pi}\left(1 - e^{-\lambda x} + \alpha\frac{1 - \cos(\pi(1 - e^{-\lambda x}))}{\pi}\right), \quad x > 0,$$

  which is completed by $K(x) = 0$ for any $x \leq 0$. It is therefore a member of what we may call the NTU family of distributions. A comprehensive study of this family is beyond the scope of the present paper.

4. Numerical Estimation

We now consider a statistical scenario where the parameter α is unknown and is estimated via the maximum likelihood (ML) method. Let n be a positive integer representing the sample size. Given the observations $x_1, \ldots, x_n$ of a random variable following the NTU distribution, an estimate of α is obtained by

$$\hat{\alpha} = \arg\max_{\alpha > 0} \prod_{i=1}^{n} f(x_i; \alpha),$$

where $f(x; \alpha) = f(x)$ is the PDF associated with the NTU distribution, so that

$$\hat{\alpha} = \arg\max_{\alpha > 0} \frac{1}{(1 + 2\alpha/\pi)^n} \prod_{i=1}^{n} (1 + \alpha \sin(\pi x_i)) .$$

The estimate $\hat{\alpha}$ is called the ML estimate (MLE) of α.

To verify the effectiveness of $\hat{\alpha}$, a simulation study is conducted. We consider samples of sizes $n = 100$, 200, 300, 400 and 500 generated via the Newton-Raphson strategy. In over $N = 1,000$ Monte Carlo runs, mean, bias, relative bias (RBias), mean squared error (MSE), root mean squared error (RMSE), average length (AIL) and coverage probability (CP) are determined using BFGS in R optim. Random samples from the NTU distribution are generated using a numerical evaluation of the quantile function: $F^{-1}(u) = Q(u)$, where $u \sim U(0, 1)$ represents random draws from the uniform distribution. The initial parameter (true) values used for data generation are the following: $\{0.25, 0.5, 0.75\}$.

Thus, the MLE is evaluated using the following performance measures:

- Mean: The average of estimated parameter values over all iterations.
- Bias $= |(1/N) \sum_{j=1}^{N} \hat{\alpha}_j - \alpha_0|$, where α_0 is the true parameter value.
- RBias $= |(1/N) \sum_{j=1}^{N} \hat{\alpha}_j - \alpha_0| / \alpha_0$.
- MSE $= (1/N) \sum_{j=1}^{N} (\hat{\alpha}_j - \alpha_0)^2$.
- RMSE $= \sqrt{\text{MSE}}$.

- AIL of 95% confidence intervals based on the normal approximation.
- CP: Percentage of times the true parameter α_0 belongs to the constructed 95% confidence intervals.

Table 1. Numerical results at $\alpha = 0.25$ using the MLE

n	Mean	RBias	Bias	MSE	RMSE	AIL	CP%
100	0.3429	0.3715	0.0929	0.2478	0.4978	1.3019	95.3676
200	0.2944	0.1777	0.0444	0.1151	0.3392	0.9538	95.5000
300	0.2836	0.1345	0.0336	0.0673	0.2595	0.7881	96.3000
400	0.2747	0.0986	0.0247	0.0533	0.2308	0.7246	96.1000
500	0.2574	0.0294	0.0074	0.0374	0.1933	0.6361	96.4000

Table 2. Numerical results at $\alpha = 0.50$ using the MLE

n	Mean	RBias	Bias	MSE	RMSE	AIL	CP%
100	0.7081	0.4162	0.2081	0.5714	0.7559	2.1332	94.9900
200	0.5858	0.1716	0.0858	0.2228	0.4720	1.4960	95.7000
300	0.5580	0.1160	0.0580	0.1244	0.3527	1.2402	96.6000
400	0.5432	0.0863	0.0432	0.0838	0.2895	1.1046	95.7000
500	0.5323	0.0647	0.0323	0.0680	0.2607	1.0147	96.1000

Table 3. Numerical results at $\alpha = 0.75$ using the MLE

n	Mean	RBias	Bias	MSE	RMSE	AIL	CP%
100	1.0113	0.3484	0.2613	1.0377	1.0187	2.9420	95.7661
200	0.8222	0.0963	0.0722	0.2975	0.5454	1.8823	96.1000
300	0.8059	0.0745	0.0559	0.1700	0.4123	1.6022	95.7000
400	0.7938	0.0584	0.0438	0.1236	0.3515	1.3680	95.7000
500	0.7832	0.0443	0.0332	0.1013	0.3183	1.2414	96.3000

The simulation results in Tables 1, 2, and 3 demonstrate that the MLE performs well in estimating the parameters of the NTU distribution. The reasons are detailed below.

- The MSE and RMSE values decrease as n increases.
- Bias and RBias decrease as n increases.
- AIL decreases with increasing n.

Overall, the MLE produces a reliable and efficient estimate for α.

5. Applications and Empirical Modeling

This section investigates the practical applicability and statistical flexibility of the NTU distribution through two real-world datasets arising from distinct scientific domains, namely economics and medical survival analysis. The empirical performance of the NTU distribution is compared with several competitive bounded and truncated distributions, including the TWD, UW distribution, TNXLD, TMED and TPLD mentioned in the introduction.

The model comparison is carried out using several classical goodness-of-fit (GoF) measures, namely the Kolmogorov–Smirnov (KS) statistic, the Cramér–von Mises (CvM) statistic, and the Anderson–Darling (AD) statistic, together with their associated p-values. Parameter estimation for all competing models was performed using the ML method.

Since the NTU distribution and all competing distributions are defined on the standard unit interval [0, 1], the original observations are transformed using the Min–Max normalization procedure. For any $i = 1, \dots, n$, the transformed value z_i of x_i is defined as

$$z_i = \frac{x_i - \min(x_1, \dots, x_n)}{\max(x_1, \dots, x_n) - \min(x_1, \dots, x_n)}.$$

To avoid numerical instability during the optimization process and prevent singularities at the boundaries, the transformed observations are slightly adjusted such that $10^{-6} \le z_i \le 1 - 10^{-6}$. This transformation preserves the relative structure of the original data while ensuring compatibility with the support of the candidate bounded distributions.

5.1. Application I: UK Food Chain Productivity Data

The first dataset consists of annual Total Factor Productivity (TFP) observations for the United Kingdom food chain sector from 2000 to 2019. This dataset was previously analyzed by Alyami et al. [3] and is widely used in studies related to economic productivity and efficiency analysis.

The original observations are given by

100.0, 99.9, 98.5, 100.1, 101.9, 101.4, 103.1, 103.2, 104.2, 102.9,
104.1, 104.8, 104.7, 105.8, 103.4, 104.1, 105.5, 107.2, 108.6, 109.0.

After applying the Min–Max transformation, the data become

0.1428571, 0.1333333, 0.0000010, 0.1523810, 0.3238095, 0.2761905, 0.4380952, 0.4476190,
0.5428571, 0.4190476, 0.5333333, 0.6000000, 0.5904762, 0.6952381, 0.4666667, 0.5333333,
0.6666667, 0.8285714, 0.9619048, 0.9999990.

Table 4 summarizes the MLEs together with the corresponding GoF statistics for all fitted models.

The results reported in Table 4 indicate that the NTU model provides the best overall fit among all competing models. In particular, the NTU model achieves the smallest KS statistic (0.1332) and the highest KS p-value (0.8698), demonstrating an adequate agreement with the empirical distribution.

Table 4. MLEs and GoF statistics for the UK Food Chain dataset.

Model	KS	KS-pval	CvM-Stat	CvM-pval	AD-Stat	AD-pval	$\hat{\alpha}$	SE($\hat{\alpha}$)	$\hat{\theta}$	SE($\hat{\theta}$)
NTU	0.1332	0.8698	0.0528	0.8618	1.3263	0.3238	0.6985	1.1590	–	–
TWD	0.1671	0.6318	0.1147	0.5214	1.1467	0.2881	4.8183	2.4603	0.2245	0.1207
UW	0.2476	0.1721	0.2677	0.1672	1.5708	0.1607	0.9639	0.2250	0.6353	0.1093
TNXLD	0.1490	0.7663	0.0844	0.6723	1.3851	0.2064	0.8767	1.5237	–	–
TMED	0.1347	0.8611	0.0553	0.8490	1.9245	0.1017	0.3470	0.1068	–	–
TPLD	0.1561	0.7140	0.1050	0.5652	1.3707	0.2921	7.3211	3.2998	0.2797	0.1295

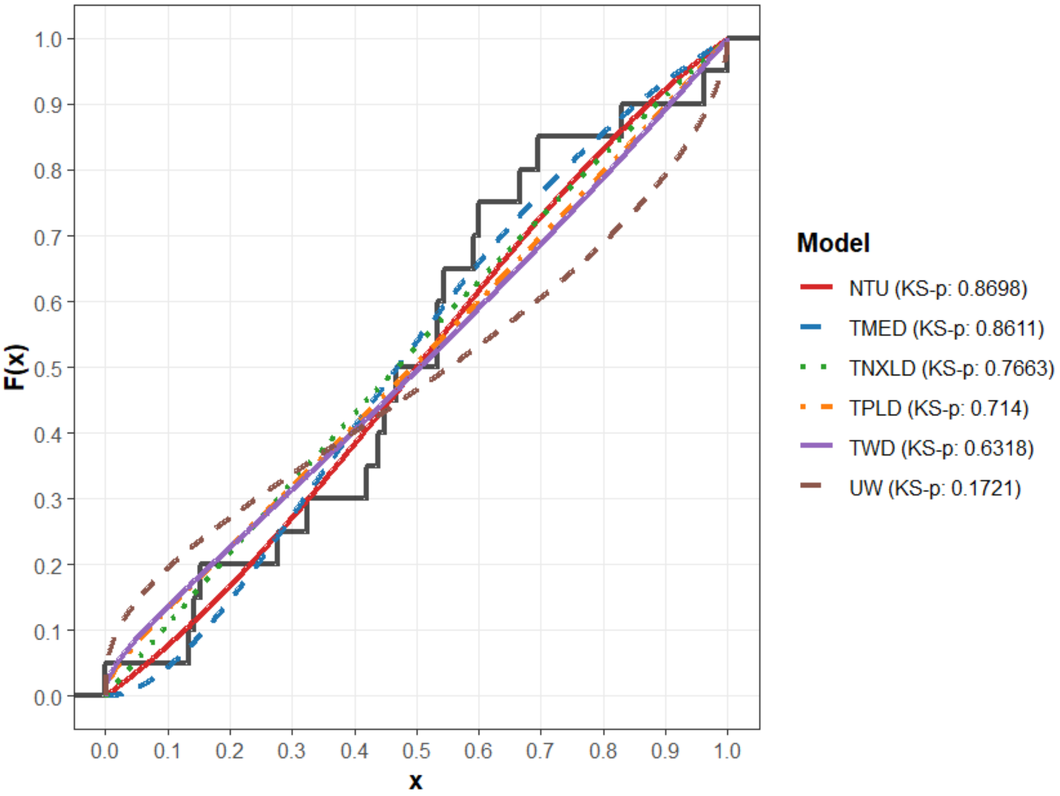

Figure 3. Estimated CDFs for the competing models for the UK Food Chain dataset.

Moreover, the NTU model also yields the smallest CvM statistic and one of the highest associated p-values, indicating superior fitting performance over the entire support of the data. Although the TMED model produces a comparable KS statistic, its AD statistic is substantially larger, suggesting inferior tail-fitting behavior relative to the NTU model.

To provide further visual assessment of model adequacy, the estimated CDFs and probability–probability (PP) plots for all competing models are presented in Figures 3 and 4, respectively. The fitted NTU model closely follows the empirical distribution over the entire support in capturing the distributional characteristics of the productivity data.

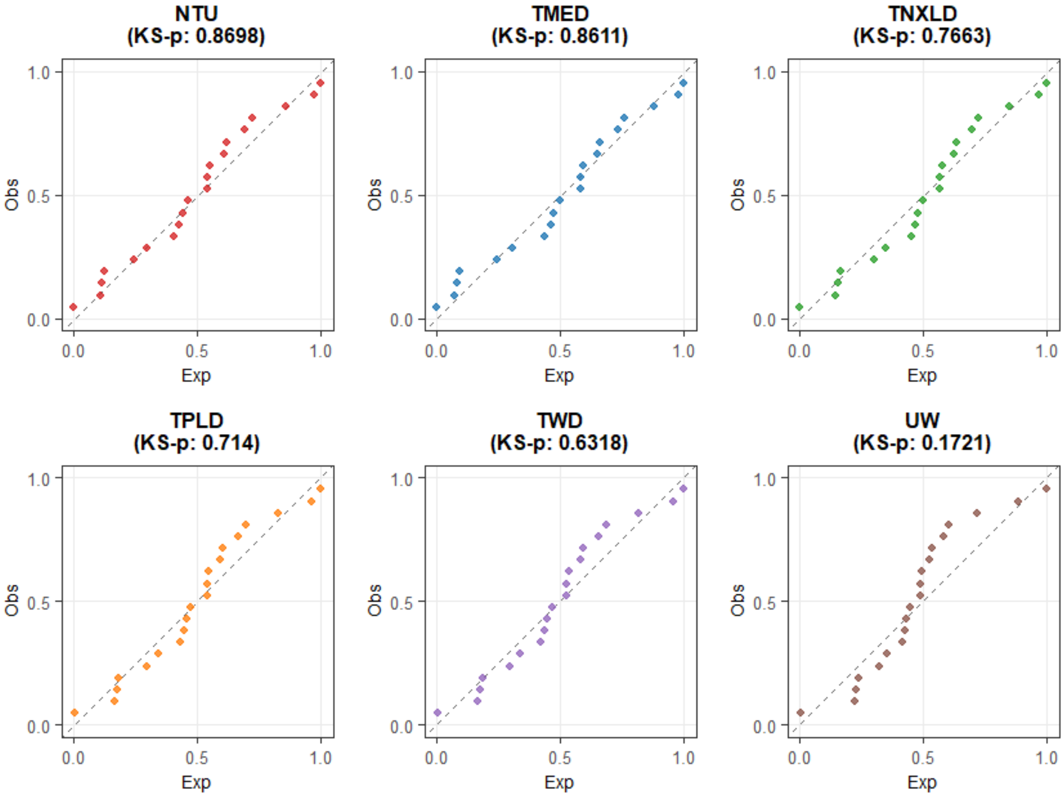

Figure 4. PP plots for the competing models for the UK Food Chain dataset.

5.2. *Application II: Leukemia Survival Data*

The second dataset consists of survival times (in months) for 20 patients diagnosed with Acute Myeloid Leukemia. This dataset was originally reported by Afify et al. [1] and has been extensively used in reliability and biomedical survival studies.

The original survival observations are given by

$$2.226,\ 2.113,\ 3.631,\ 2.473,\ 2.720,\ 2.050,\ 2.061,\ 3.915,\ 0.871,\ 1.548,$$
$$2.746,\ 1.972,\ 2.265,\ 1.200,\ 2.967,\ 2.808,\ 1.079,\ 2.353,\ 0.726,\ 1.958.$$

The corresponding transformed observations are

$$0.4703669,\ 0.4349326,\ 0.9109439,\ 0.5478206,\ 0.6252744,\ 0.4151772,\ 0.4186265,\ 0.9999990,$$
$$0.0454688,\ 0.2577611,\ 0.6334274,\ 0.3907181,\ 0.4825964,\ 0.1486359,\ 0.7027281,\ 0.6528692,$$
$$0.1106930,\ 0.5101913,\ 0.0000010,\ 0.3863280.$$

The MLEs and GoF measures for the fitted models are summarized in Table 5.

From Table 5, it is evident that the NTU distribution again provides the best overall fit according to most GoF criteria. Specifically, the NTU model attains the smallest KS statistic (0.1717) and the largest KS p-value (0.5407) among all fitted models, indicating strong compatibility with the empirical survival data.

Table 5. MLEs and GoF statistics for the Leukemia Survival dataset.

Model	KS	KS-pval	CvM-Stat	CvM-pval	AD-Stat	AD-pval	$\hat{\alpha}$	SE($\hat{\alpha}$)	$\hat{\theta}$	SE($\hat{\theta}$)
NTU	0.1717	0.5407	0.0780	0.6830	1.2361	0.3474	0.7578	1.1699	–	–
TWD	0.1841	0.4530	0.1517	0.3870	1.2739	0.2407	4.6101	2.5956	0.2107	0.1243
UW	0.2608	0.1092	0.2926	0.1416	1.7027	0.1351	0.8904	0.2114	0.6604	0.1134
TNXLD	0.2052	0.3235	0.1145	0.5223	1.4780	0.1819	1.4982	1.2284	–	–
TMED	0.1898	0.4156	0.0892	0.6456	2.1680	0.0750	0.2969	0.0794	–	–
TPLD	0.1862	0.4391	0.1427	0.4154	1.2581	0.2461	6.9637	3.4219	0.2640	0.1326

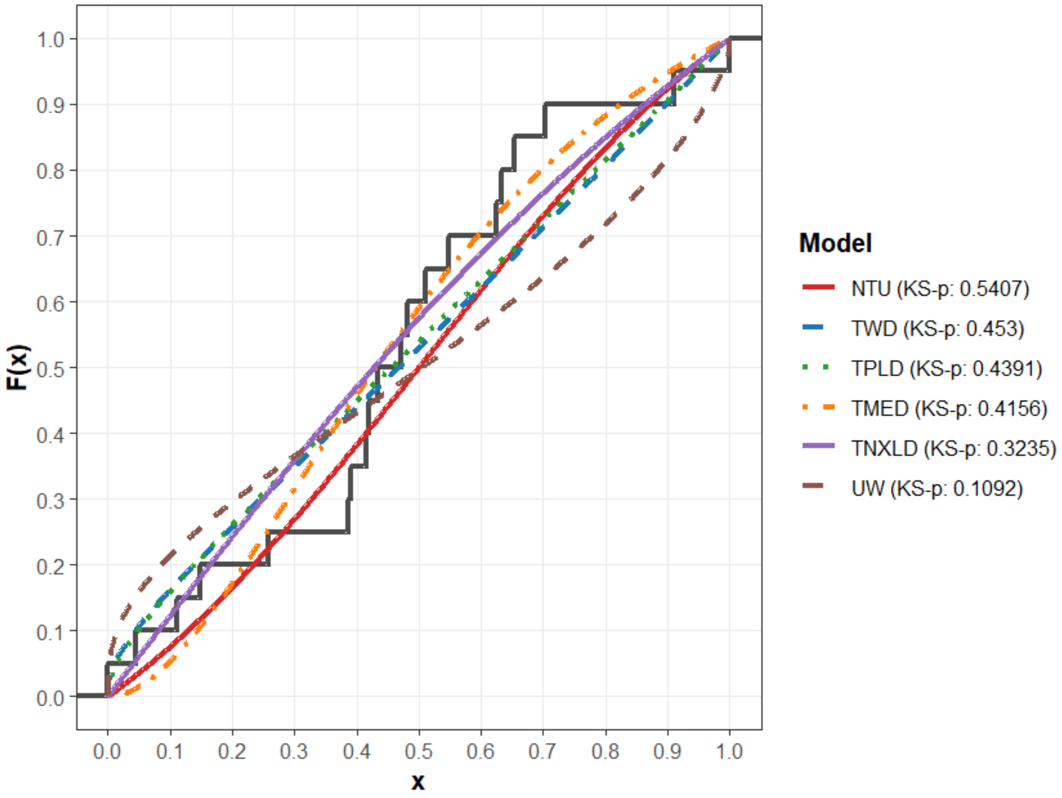

Figure 5. Estimated CDFs for the competing models for the UK Food Chain dataset.

In addition, the NTU distribution produces the smallest CvM statistic and competitive AD values, highlighting its ability to accurately model both the central behavior and tail structure of the dataset. By contrast, the UW distribution exhibits comparatively poorer performance, reflected by larger discrepancy measures and lower p-values.

Figures 5 and 6 display the fitted CDFs and PP plots. The visual evidence further supports the robustness and flexibility of the NTU distribution for modeling bounded biomedical survival data.

The empirical investigations conducted on the two real-world datasets clearly demonstrate the strong modeling capability and flexibility of the NTU distribution. Across both applications, the NTU model consistently outperforms the competing distributions according to the majority of the GoF measures.

For the UK Food Chain dataset, the NTU model achieves the smallest KS and CvM statistics together

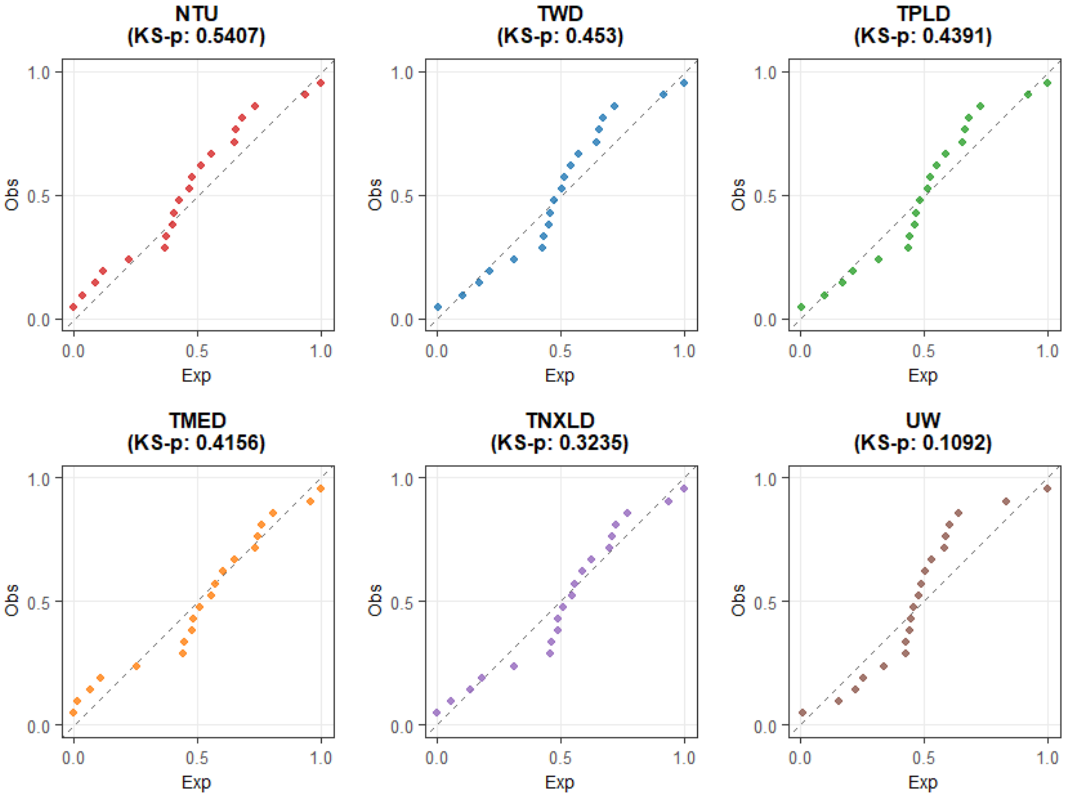

Figure 6. PP plots for the competing models for the UK Food Chain dataset.

with the largest associated p-values, indicating excellent agreement with the observed economic productivity data. Similarly, for the Leukemia Survival dataset, the NTU model maintains its superiority by producing the best overall fit among the considered models.

The graphical analyses based on the estimated CDFs and PP plots also reveal that the NTU model closely follows the empirical behavior of the datasets over the entire support.

6. Conclusion

In this paper, we introduced a new trigonometric unit distribution, the NTU distribution, that generalizes the uniform distribution through a single shape parameter. The main statistical properties of the proposed model, including its CDF and mean, were derived and discussed. The numerical analyses showed that the NTU model provides considerable flexibility and achieves competitive performance when compared with existing alternative models. Applications to two real data sets further demonstrated the practical usefulness and effectiveness of the NTU model in analyzing data into $[0, 1]$. A natural direction for future research is the further exploration of the NTU family of distributions, as well as the use of the NTU distribution as a baseline for constructing broader and more flexible distributions. We may also think of the contruction of a regression model.

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